The Effects of Confinement on Explosive Detonation in Boreholes

Presented by ICI Explosives

1 Introduction

The problem considered by the study group was to determine the structure and speed of a detonation wave propagating down a cylindrical borehole and to find how the speed depends on the properties of the rock in which the borehole is drilled. This problem is analogous to the description of detonation within a "rate-stick" ([1], p. 64) which consists of a long cylindrical explosive in a tube, sometimes providing little confinement of the explosive at its edges; a useful pair of references describing unsteady behaviour under these conditions is [2] & [3]. The main reference that was found to cover the fundamentals of this type of phenomenon (especially with strong confinement) was [4] and much of the study group was spent in attempting to unravel the highly condensed information contained in this important paper.

2 Basic structure of the detonation wave

From experimental work it is known that the detonation wave travels at a constant velocity soon after being initiated so that a steady travelling wave-structure was considered to be appropriate. Taking the speed of the wave to be V, the value of V becomes an eigenvalue of the problem. The physical system has two distinct but coupled parts, relating firstly to the chemical reaction and the motion it induces in the explosive and secondly to the motion of the rock in response to pressure changes in the explosive.

2.1 The Explosive

The most useful set of equations to be solved in the explosive is the reactive Euler model, written here in variables moving with the constant speed of propagation V:

$$V\frac{\partial \rho}{\partial z} + \nabla \cdot (\rho \bar{u}) = 0$$

$$V\frac{\partial \bar{u}}{\partial z} + (\bar{u} \cdot \nabla)\bar{u} = -\frac{\nabla p}{\rho}$$

$$V\frac{\partial e}{\partial z} + (\vec{u}\cdot\nabla)e = \frac{p}{\rho^2}(\vec{u}\cdot\nabla)\rho - q\dot{\lambda}$$

$$V\frac{\partial \lambda}{\partial z} + (\vec{u}\cdot\nabla)\lambda = \dot{\lambda} = -\lambda^n k(p,\rho)$$

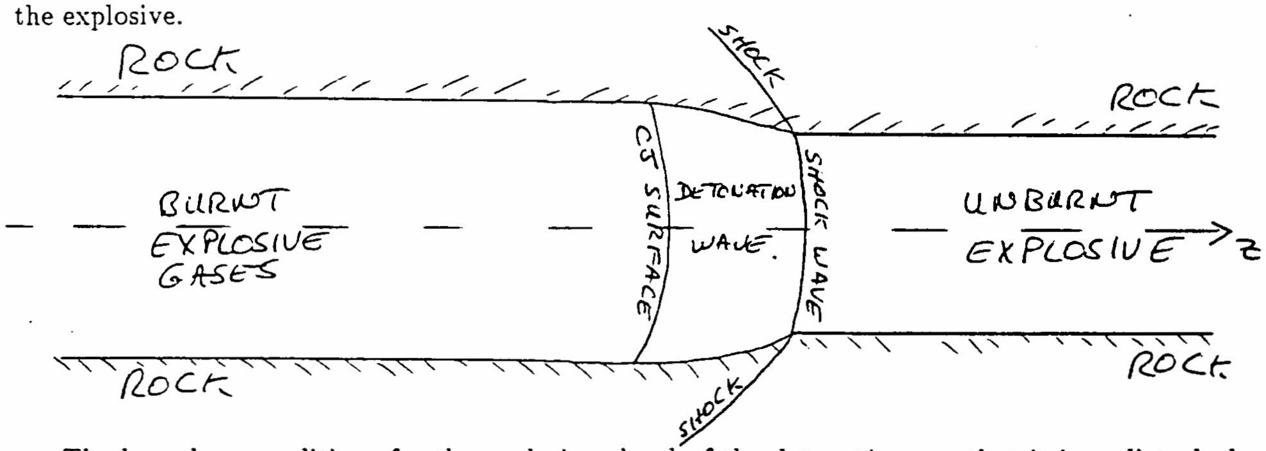
together with an equation of state for the gas. An example is the simple polytropic law

$$e = \frac{1}{\gamma - 1}\frac{p}{\rho}.$$

This is equivalent to an ideal gas law, although it is common to take values of the polytropic coefficient $\gamma = C_p/C_v$ to be three or more when modelling the behaviour of the dense gases that are produced during the detonation.

In these equations: q is the total amount of energy per unit mass that is available within the chemistry before reaction begins; λ measures the relative amount of energy remaining bound up in the chemistry at any stage; $k(p,\rho)$ represents a kinetic rate-law for the chemical reaction which (for a simplified overall reaction) is taken to be of order n; and z is a coordinate taken along the axis of the borehole. The solution structure has been assumed to be axisymmetric. It may be easier to solve this problem in streamline coordinates, as proposed in [5], because this could make some aspects of the numerics easier (see later).

When a detonation wave propagates down a cylindrical borehole there are two crucial surfaces within the flow of explosive that need to be considered. The first is the shock wave that propagates into the ambient explosive and the second is the CJ (Champan–Jouget [1]) surface which arises when the burning explosive expands sufficiently for the flow to become supersonic relative to the shock that precedes it. Between these two surfaces the flow is subsonic while the supersonic nature of the flow behind the CJ surface ensures that information from this region never penetrates the CJ surface to influence the flow ahead of it. The figure gives the basic structure envisaged within the explosive.



The boundary conditions for the explosive ahead of the detonation are that it is undisturbed so, in the moving coordinate frame explosive, enters the detonation wave at a uniform velocity of speed V in the z direction, $\vec{u} = (0,0,V)$; it is unburnt at this stage so that $\lambda = 1$; the pressure and density are known and are such that chemical equilibrium prevails, $k(p,\rho) = 0$. Applying Rankine-Hugoniot jump conditions at the curved shock surface gives velocity, pressure and density behind the shock (note the shock must be strong enough to ensure that behind the shock $k(p,\rho) \neq 0$).

A second boundary condition comes form the fact that the borehole is of finite extent. Therefore, the velocity of the gas goes to zero at this point. This condition forces the problem to retain some unsteadiness behind the CJ surface (more later); however, because information from this region can never penetrate the CJ surface we can anticipate that the crucial shock–CJ region will remain steady and essentially decoupled from any unsteadiness that occurs a long way behind it. That is, after any initial transients produced during its initiation, the detonation will remain underdriven rather than overdriven by the rear boundary condition [1]. The properties of the detonation wave are therefore independent of this boundary condition.

The gas flow within the borehole is then determined by considering the interaction with the rock which is specified by considering conditions at the borehole wall. At this unknown position $r = R(z)$, the pressure in the gas will be the normal stress in the rock and the normal velocity of the surface will be the normal velocity of the explosive. The value of $R(z)$ is a known constant ahead of the shock.

2.2 The Rock

There was substantial discussion as to what types of model were appropriate for modelling the rock. The stress levels within the rock may be near or even well above its compressive strength. This would depend crucially on the nature of the rock as well as on the peak pressures attained within the detonation wave. Indeed conditions both above and below the plastic limit may arise in different regions since the pressure waves produced by the explosive would be rapidly attenuated as they propagate away from the borehole.

Two possible models were therefore considered. In the first model, as is common in studying condensed explosives, the rock may be taken to behave as a dense nonreactive compressible fluid satisfying a polytropic equation of state (gas-law with a γ value somewhere around 3 or more). Thus the rock would be treated in exactly the same way as the explosive (apart from the essential fact that it remains inert). The second possible model was to consider only the compressive waves in the rock and to take the wave speed as its elastic value.

If one adopts the polytropic equation of state, then the question of actually prescribing a appropriate values for γ and the reference pressures and densities in the rock gave rise to some debate since it is not obvious whether the rock's wave speed should drop below its elastic value once it has been compressed beyond its plastic limit, or whether the wave speed should increase monotonically. In particular the elastic model need not be (and probably is not) a linearised limit of the polytropic model. On balance, it was felt that other criteria should probably be used in estimating γ. Indeed, the same dichotomy arises in prescribing γ for the explosive.

Of course, "far" from the detonation wave the elastic law is appropriate, so there must be a transition region from the polytropic model. Although this issue was not resolved, it may not present a major practical difficulty. The enormous pressures found within the shock–CJ structure of the detonation wave can be several orders of magnitude greater than the pressures behind the CJ surface. Thus (for typically narrow detonation waves) only a relatively small region of the rock might be driven beyond its yield stress. This small region would, nevertheless, also have a profound effect on the structure of the detonation wave. At larger distances (on the scale of the borehole length, for example) this region, and indeed the thickness of the detonation wave itself, would be tiny. The overall jumps through the detonation wave could then be used to provide local jump conditions to be used within a linear elastic model for the behaviour of the rock (see below).

If the nonreacting gas-type model is taken to be appropriate, then the equations satisfied are the same as those governing the explosive except that there is of course no chemical reaction in the surrounding rock (equivalent to setting $\lambda \equiv 0$ within the rock); other constants, such as γ and the density ahead of the shock may be different. The pressure and velocity ahead of the shock should be the same in the rock as in the explosive.

A paper was discussed [5] that outlined some work that was intended to be done by modelling the rock as a dense gas and the explosive as a reacting gas (as above) but this paper represents no more than a statement of intention. It offers no details of the results either known or anticipated by the authors; we have no firm information on the progress of this project. As mentioned earlier, however, the paper does give an alternative formulation of the model using coordinates that follow the flow through the detonation wave. This may prove very useful in any numerical study as it would automatically accommodate the rock–explosive boundary which would otherwise have to be "fitted" numerically, a difficult and error-prone task.

The rock model couples with the explosive model at the borehole wall through the continuity of pressure and the kinematic condition (continuity of normal velocity) so that, ultimately, neither portion can be solved independently of the other.

3 Analysis of the shock/CJ region

It is well known that if the rock does not move as the detonation wave passes then the detonation wave propagates at its CJ speed and if the rock does move then the overall curved detonation structure propagates more slowly. In the case of a perfectly immobile rock the shock is planar and the CJ surface is found at the end of the chemical reaction (which may be away at infinity).

There is a classic paper [6] which considers the shock to be nearly planar (large radius of curvature) and the reaction rate to be large. This second condition, together with the expansion brought on by the curvature of the shock, brings the CJ curve close to the shock so that the entire detonation wave structure becomes nearly planar. These assumptions are used in [6] to reduce the problem to one of solving only for the detonation structure along the axis of symmetry of the explosive. One of the results of the curved shock and subsequent reaction is that the flow coming out of the CJ curve is deflected at an angle to the shock/CJ surface directions.

The paper by Bdzil [4] extends this approach by considering more detailed effects of shock curvature. Most significantly, Bdzil extends his analysis away from the axis of symmetry to cover the entire region where the shock–CJ structure can be considered to be thin. The analysis gives the detonation wave structure in terms of solutions to an ordinary differential equation for the flow deflection as a function of distance from the axis of symmetry on the surface of the (thin) detonation wave. The problem is then closed by specifying the deflection produced by the detonation close to the inert rock walls. This boundary condition on the ODE represents the confinement conditions due to the rock and thereby determines the structure, the speed, or even the failure, of the detonation wave.

Bdzil's main result in [4] appears to be that, by considering the problem of rock motion arising from the detonation waves, ie. the travelling pressure distribution between a weakly curved shock and the CJ curve (extended, by assumption, right up to the rock boundary), the angle of deflection δ of the rock interface can be determined. The model is then closed by insisting that the explosive at the borehole wall, as described by Bdzil's extension of the analysis in [6] leaves the quasi-planar detonation at this same angle δ. It is this assumption of equality of these two angles that is central to the model. Except possibly where the rock is very stiff and almost completely confining this is a weakness; significant local alterations in the chemistry should be anticipated as energy is lost into a compliant rock surface so that Bdzil's description of the quasi-planar detonation should fail locally. The assumptions underlying this crucial aspect are not made explicit in [4] and are acknowledged [3] to be a weakness. Describing the fully two-dimensional region in which the detonation meets the rock (or any compliant surface) remains a crucial open question at this time.

Bdzil's paper [4] is written in a highly condensed fashion making it difficult to follow in detail. Because of the difficulty in interpreting the paper it was felt that only by looking at the analysis of Bdzil in greater depth using a more fully expanded mathematical treatment could the reasoning and analysis be properly clarified.

The key problem of addressing the nature of the two-dimensional boundary layer where the shock meets the wall (and the quasi-planar assumptions of the detonation fail) remains to be solved, except possibly in the strongly confining limit where Bdzil assumptions may continue to be valid.

4 Analysis of the borehole downstream of the shock / CJ region

The work of Bdzil (however it may be clarified or extended) gives the speed at which the detonation travels, but there was also discussion of how the borehole responds behind the shock/CJ region. Because the pressures in this region are much lower the rock can be modelled as allowing elastic compressive waves with constant wave speed. For the cases of interest this rock wave speed is less than the detonation speed and that the displacement of the rock is relatively small compared to the radius of the borehole. The mathematical problem in the rock therefore reduces to supersonic small-disturbance theory. In practice it is necessary to consider the radial problem with the small disturbances occurring on the finite radius representing the undisturbed borehole wall, but much insight would be gained by looking at the more classical two dimensional version.

The novelty in the problem is that instead of having the rock surface specified, as would usually occur with a wing profile, this must be determined by considering the flow of the (now burnt) explosive. At several borehole radii from the detonation wave the flow of the explosive can be taken as one dimensional and the flow is supersonic in this region (relative to the moving shock) so the coupled problem is posed by imposing continuity of pressure and the kinematic condition at the borehole wall. No further analysis has been done on this problem.

5 Areas to be pursued

From the discussions within the week the following areas of study were identified as important to understanding the detonation / rock interaction problem.

- Because of the importance of the paper by Bdzil [4] it was felt that this work should be studied in greater depth, most specifically with the intention of re-working Bdzil's analysis in a clearer and more easily understandable form. The appropriate procedure is to put the approximations made in the paper into a systematic structure using the method of matched asymptotic expansions. This will highlight both the assumptions that must be made to use the analysis of Bdzil and the limitations of that analysis. A number of more expository articles are available ([2], [3] & [7]) that should help in this task.
- The local two-dimensional problem that describes the interaction of the detonation with the rock needs to be worked out properly. This may well only be accessible as a numerical task, but it does provide the essential link that would allow Bdzil's analysis of quasi-planar detonations to be linked in with more general classes of confining material. It may be possible to extend such a two-dimensional numerical problem to one in which the entire detonation structure is solved numerically. A very useful reference for the numerical aspects of such a task with unsteady flow is [8].
- The latter objective is the one proposed in [5], so that the development of this work should be followed closely. At the moment it is not clear that this project is being pursued actively.
- The simplified analytical models should be coded numerically to develop usable predictive models of the detonation velocity given suitable detonation deflections near the edges of the explosive.
- The problem involving coupled small-disturbance supersonic rock & burnt explosive flow for a given detonation speed with given jump conditions needs to be considered in more detail.

References

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