PROBLEM 3

AN INVESTIGATION OF DIE DESIGNS FOR TUBE DRAWING

1. OUTLINE OF PROBLEM

This problem was suggested to the Mathematics-in-Industry Study Group by Metal Manufacturers who produce, among other goods, a wide range of drawn non-ferrous tubes. The process under investigation was the drawing of pipes using a die and a plug. The basic setup is sketched in Figure 1.

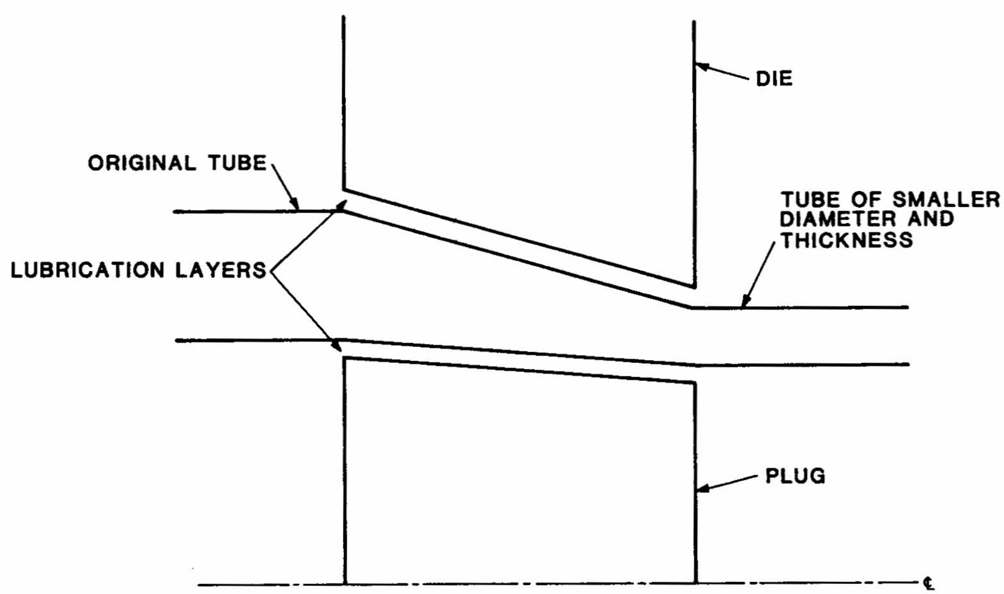

Figure 1. Illustrating the geometry of tube drawing equipment.

The problem has received considerable attention in the literature (see for example Avitzur (1968, 1982), Durban (1980)). However, most authors appear to use simple friction when modelling the shear stresses due to the die and plug.

Because the interaction between the tube, the plug and the die is crucial, it was decided to examine the role of the lubricant in the drawing process.

2. THE MODEL USED

Because of the constraints of the MISG and as the aim was to obtain qualitative rather than quantitative results, a one dimensional model was examined. In addition, it was assumed that:

i. The tube is perfectly plastic.

ii. The die and plug are rigid.

iii. Hydrodynamic lubrication theory is valid between the tube and die and between the tube and plug. In addition, the two lubrication layers were assumed to have equal thickness.

iv. The position of the die is fixed.

However, as was pointed out in the discussion:

i. One dimensional models are inadequate for estimating the redundant work and are unsuitable unless the die angle is small.

ii. Effects such as work hardening must be taken into account when considering the efficiency of drawing in tandem.

iii. The existence of a sufficiently thick layer of lubricant to sustain hydrodynamic lubrication was questioned. It was also pointed out that if such a layer did exist, the flow could well be non-Newtonian. The question of layer thickness was not fully resolved although an order of magnitude calculation based on a rough estimate of the quantity of lubricant used did suggest that the layers were of the order of 10 microns.

iv. The lubrication layers will not be of equal thickness in general but this can easily be incorporated into the model.

v. The floating plug case is of much more interest than the fixed plug case. However, if the fixed plug can be solved for an arbitrary geometry, the floating plug case can be expected to be solved from this by determining the displacement of the plug which yields zero net force on the plug.

In Figure 2 below we have indicated some of the geometrical aspects of the drawing process while Figure 3 shows some of the stresses on the tube.

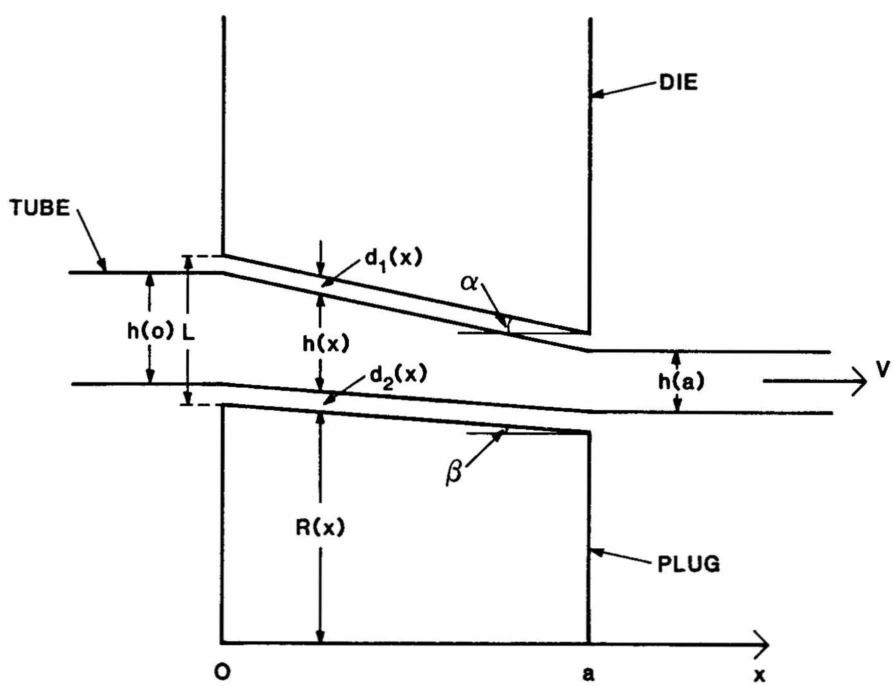

Figure 2. Tube drawing geometry and the definition of variables.

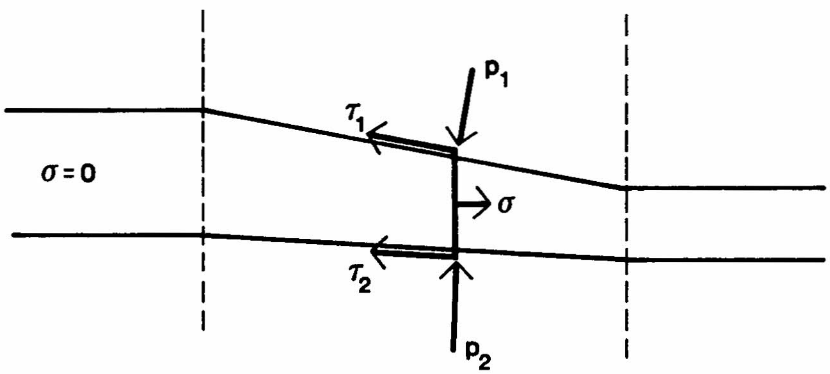

Figure 3. The stresses on the tube.

For equilibrium we require

$$\frac{d}{dx}((2R + 2d_2 + h)h\sigma)$$
$$= 2 p_1 (\tan \alpha - d_1') (R + d_2 + h) - 2 p_2 (\tan \beta - d_2') (R + d_2)$$
$$+ 2\tau_1 (R + d_2 + h) + 2 \tau_2 (R + d_2) \tag{1}$$

In addition, lubrication theory yields (when h, α and β are small)

$$\tau_i = \frac{1}{2} \frac{dp_i}{dx} d_i + \mu U_i/d_i \text{ ; } i=1,2 \tag{2}$$

and

$$\frac{dp_i}{dx} = \frac{\sigma\mu}{d_i^3} (Ud_i - \frac{Q_i}{\pi R}) \text{ ; } i=1,2 \tag{3}$$

where Q_i is the flux in the ith lubrication layer. Furthermore, continuity yields

$$U_i \approx R(a)h(a)V/Rh, \quad i=1,2. \tag{4}$$

As is customary for one dimensional models, we assume that the shear contribution to the die pressure is small and that the principal stresses can be taken to be

$$\sigma_1 = \sigma \ , \ \sigma_2 = -p.$$

Then the yield condition in plane strain is

$$\sigma + p = S = 1.15\ Y$$

where Y is the yield stress. Thus,

$$p = p_1 = p_2 = S-\sigma \tag{5}$$

Finally, we have the geometric constraint

$$h + d_1 + d_2 = r + (\tan\beta - \tan\alpha)x \tag{6}$$

If we now assume that

$$d_1 \approx d_2 \ ; \ Q_1 \approx Q_2 \ ; \ \tan\alpha,\ \tan\beta << d;$$
$$R,\ h,\ R',\ h' << d,\ d',$$

then equations (1-6) yield equations of the form

$$f_1\ (p, \frac{dp}{dx}, d, Q) = 0,$$
$$f_2\ (\frac{dp}{dx}, d, Q) = 0,$$

and in order to solve these equations we need two auxiliary conditions. From lubrication theory, we have

$$p = 0 \text{ when } x = 0,a \tag{7}$$

while from (5) and $\sigma(0) = 0$ we obtain

$$p = S \text{ when } x = 0. \tag{8}$$

Clearly (7) and (8) contradict each other which essentially reflects the fact that the one dimensional plasticity model is not valid in a region near the end points $x = 0$ and $x = a$ (in contrast to this, lubrication theory is valid over the whole region if the layer thickness is sufficiently large and the flow is Newtonian). A possible remedy is to introduce elastic relief regions in the neighbourhood of these points.

However, in the interior of the interval, the one-dimensional model may still give some useful insight. If, in equation (1), we assume that

$$h,\ d,\ \tan\alpha - \tan\beta \leq R,$$

$$\tan\alpha,\ \tan\beta >> d',$$

we obtain

$$\frac{d}{dx}(h\sigma) = -p\frac{dh}{dx} + 2\mu U/d$$

which reduces, with the aid of (5), to

$$h\frac{dp}{dx} = S\frac{dh}{dx} - 2\mu U/d. \qquad (9)$$

On combining (9) and (3) we get

$$S\frac{dh}{dx} - 2\mu U/d = \frac{\sigma\mu h}{d^3}\left(Ud - \frac{Q}{\pi R}\right)$$

which can be solved and yields a result of the form

$$d = d(x,Q).$$

Of course, Q is unknown but can in principle be measured. Such a measurement would determine whether the lubrication layer is sufficiently large.

In most applications of lubrication theory, the pressure is much larger than the shear stress. We expect this is the case here also. Then (9) reduces to

$$h\frac{dp}{dx} = S\frac{dh}{dx}$$

which has the solution

$$p = S\log(Ch(x))$$

As $p \approx S$ when x is small, this gives the approximation

$$p \approx S\log(eh(x)/h(0))$$

3. CONCLUDING REMARKS

The use of a one-dimensional model to investigate die design for tube drawing is unsatisfactory from several points of view. At best it is valid only in the central part of the die. Because the geometry at the ends of the die will play a crucial role in determining the nature of the lubrication layer, it is essential that these regions be modelled correctly. In addition the one-dimensional model outlined is reasonable only for close pass drawing. Where there are significant stress variations across the tube, there will be considerable relative straining of the material in a cross-section. It would seem worth investigating generating strainings where the stress state was as uniform as possible across the section so as to reduce the amount of work hardening produced. A possible philosophy of design is as follows:

1. Calculate a surface traction distribution to minimize strain hardening.
2. Determine the lubrication layer thicknesses consistent with this loading.
3. Determine the die shapes which will give the best results.

Prof. Mahony and Dr Fowkes of the University of W.A. and Dr Coleman of the University of Wollongong have indicated a willingness to pursue the problem.

REFERENCES

Avitzur, B., Metal forming; processes and analysis (McGraw Hill, 1968).

Avitzur, B., "Study of flow through conically converging dies", Wire Ind. (Jul. 1982), 449-619.

Durban, D. "Drawing of tubes", Trans. ASME 47 (1980), 736-740.