PROBLEM 5

DERIVATION OF CONSTITUTIVE EQUATIONS FOR MACROSCOPIC STRESS-STRAIN PROCESSES AND THE INTERPRETATION OF SEISMOGRAMS IN UNDERGROUND COAL MINING

1. INTRODUCTORY COMMENTS

Two problems were examined for the Australian Coal Industry Research Laboratories (ACIRL). They were

ACIRL #1: Derivation of constitutive equations defining macroscopic stress-strain processes in rocks.

and

ACIRL #2: Inversion of seismogram data in the interpretation of the wave guide characteristics of coal seams.

At the initial presentation of the problems on the Monday, the background to and motivation for the first problem was given by ACIRL's representative, Dr Peter Hornby.

Though the questions it raised are important industrially, the problem was very difficult because of its speculative nature and the large number of possible attacks on it that could be made. The organisers of the Study Group were not sure that suitable people would be present to work on the problem at the meeting, hence another problem (ACIRL #2) was also prepared in case the discussion on ACIRL #1 turned out to be sterile.

Although more than 10 delegates participated in the initial discussions of ACIRL #1 on the Tuesday morning, it was quickly appreciated that there was greater interest in the second problem. Abstracts for both had been circulated in advance to attendees. The presentation for ACIRL #2 was, therefore, given by Dr Peter Hornby before lunch on the Tuesday. Thereafter, attention concentrated on ACIRL #2, although informal discussions on ACIRL #1 occurred regularly throughout the week.

2. ACIRL #1

2.1 Background and motivation

One of the activities of ACIRL is to assess, for different coal mining groups, the consequences of various mining operations at locations either being mined or being considered for mining. The aim is to predict how the mining operation should proceed to ensure excavation stability, to control the onset of non-recoverable deformation of the shaft, to minimize surface subsidence, etc. But, such assessments can only be reduced to a routine activity if the whole process is computerized so that the changing geology with location can be modelled rapidly and semi-automatically.

To achieve such a goal, the first step is to derive the appropriate constitutive equations. It is within the context of their derivation that ACIRL's first problem arises.

Because the mining operation is on a scale which is orders of magnitude larger than that of the experimentally derived constitutive laws for the relevant stress-strain relationship, it is necessary to modify and supplement these experimental and microscopic results in order to obtain constitutive equations which apply on the macroscropic scale of the mining operation. Because fracturing and faulting must be allowed for when modelling a mining activity, it is not simply a matter of constructing, on a suitably fine grid, an algorithm based on the experimental stress-strain relationship. The fracturing and faulting must be built into the computational process in some explicit way.

In ACIRL's situation, such an approach is not a viable proposition. The requirement to work with modest computer resources forces the need to work computationally on a macroscopic scale representative of the mining operation; and, thereby, poses the key question: how should the experimental laws, the fracturing and faulting processes, and other appropriate factors be combined to generate constitutive laws appropriate for modelling the mining operation on the macroscopic scale?

In the Abstract for this problem circulated prior to the Mathematics-in-Industry Study Group, the possibility was suggested of using statistical (stochastic) modelling to derive the required equations.

2.2 A summary of MISG deliberations

From the formulate→solve→interpret point of view of applied mathematics, the examination of ACIRL #1 reduced to essentially a process of formulation. Deliberation therefore concentrated on a discussion of alternative strategies by which the desired constitutive equations could be derived. They included:

1. An examination of the approach taken by the Oxford Study Group when they considered a similar problem. This material was supplied by Dr Chris Coleman.

2. The utility and limitations of the blocking methods, which were examined in some detail. Such methods are used by structural engineers to predict faulting behaviour in the walls of mine shafts.

The walls of the shaft are partitioned into large macroscopic blocks which are then moved according to pre-determined rules and assumed initial forces. In their simplest form, only Newton's equations of motion are taken as the constitutive equations for the blocks. Deformation of the blocks is ignored. Computationally, this poses difficulties, as constraints must be invoked which ensure that the blocks move as solid bodies relative to each other. If deformation is incorporated into the modelling, then the resulting constitutive equations correspond to the discretization of an initial value problem for stiff ordinary differential equations, and behave accordingly computationally.

Efforts to modify blocking methods, in order to circumvent such difficulties, lead back to a finite element description and an examination of the questions which are at the core of the ACIRL #1 problem.

3. The potential for developing a stochastic model of the Weibull type was considered. Weibull modelled tensile failure of a specimen by considering a number of random elements in series. For compressive stress, however, there is an important non-elastic deformation prior to failure. The following crude model with the desired features was suggested. A number of elements are placed in parallel. Each is perfectly elastic with modulus E, but they fail at different compressive strains. Let $P(\varepsilon)$ denote the probability of failure at strain ε and assume that the elements do not interact. Then the total stress σ of the combined system has expected value

$$\langle\sigma\rangle = E\varepsilon\{1-P(\varepsilon)\} \sim \begin{cases} E\varepsilon & \text{as } \varepsilon\to 0 \\ 0 & \text{as } \varepsilon\to\infty \end{cases}$$

The model also predicts variation among specimens - a desirable, probably vital, feature of a useful model. Work continues on more realistic models and their comparison with data on laboratory specimens.

3. ACIRL #2

3.1 Background and motivation

The mechanization of coal mining has reached the point where, under favourable circumstances, vast quantities of underground coal can be mined inexpensively. The challenge is to locate, at minimal cost, coal deposits with favourable mining characteristics: good quality, reasonably thick and fairly uniform seams of coal with relatively strong roofs and floors extending over areas of the orders of square kilometres.

Drilling programs are used to locate seams of a suitable thickness and quality. However, because direct observation through drilling is too expensive, it cannot be used to make the more detailed assessment of seam characteristics necessary for planning mining operations. This must be done using information collected about the seam from the indirect measurement techniques of geophysical exploration.

The economic advantages associated with the collection of such indirect measurements must be offset against the greater mathematical and computational sophistication required in their analysis and interpretation. In particular, because the resulting mathematical models will be improperly posed, and the degree of ill-posedness is proportional to the indirectness of the measurements, it is crucial to use measurements which exploit the problem context to the fullest.

Because the characteristics of interest are seam geometry (possibly modified by the presence of bore holes and even the results of earlier mining operations) and the mechanical properties of the rock, reflection and transmission of seismic waves generated and observed in the seam represent the closest coupling which can be achieved between indirect geophysical measurements and the information desired.

3.2 Problem description

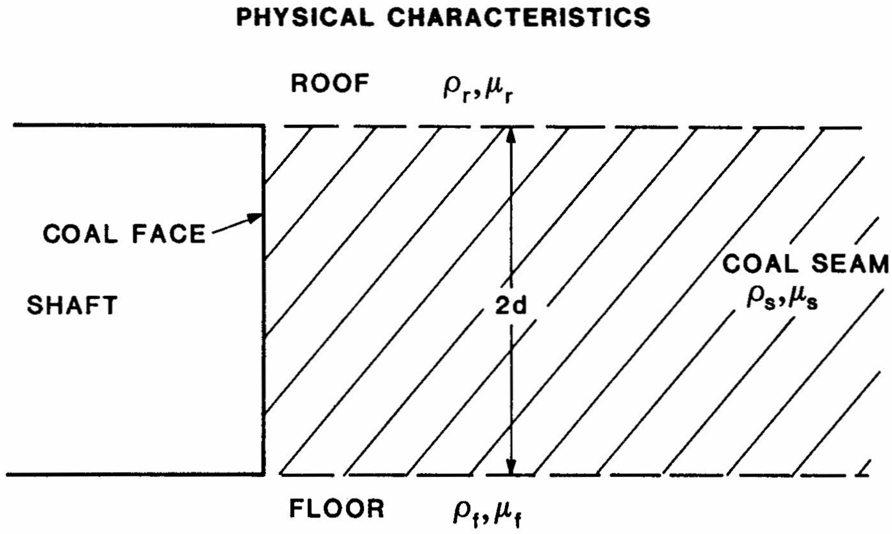
Because the elastic properties of coal are substantially different from the sandstone rock which generally forms the roofs and floors of coal seams (elastic wave velocities are higher in sandstone than in coal), the seam acts as a wave guide for the seismic (elastic) waves generated in it (see Fig. 1).

Thus, because of the mechanical nature of the seam characteristics and the existence of previously drilled bore holes (even earlier mining operations in some situations), the reflection and transmission of seismic waves generated and observed in the seam represent the closest coupling which can be achieved between indirect geophysical measurements and the seam characteristics of interest.

In transmission seismology, the elastic waves generated by an explosion in the seam are observed as seismograms or geophones placed in the available boreholes about the source of the explosion (see Figure 2). Because the coal seam acts as a dispersive medium in which the longer period waves travel faster than the shorter period ones, the observed seismograms appear as a dispersed wave train (see Figure 2). The first arrivals on the seismograms correspond to the surface waves which have mainly travelled along the roof (floor) of the seam and can therefore be used to determine the seismic wave velocities for the materials in the roof (floor). If the material in the roof and floor are the same and have homogeneous isotropic properties, then (see Figure 2)

$$\hat{V}_i = d_i/t_i, \; i=1,2,3,\ldots,$$

represent independent estimates of the SH seismic wave velocity V_{SH}, and thereby yield constraints on the possible values of μ_r and ρ_r.



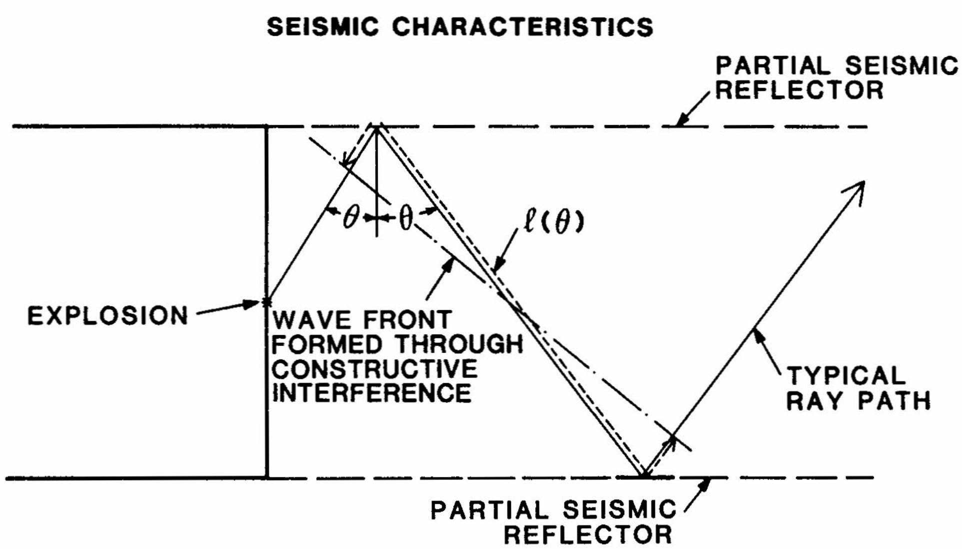


Figure 1. A rectilinear wave guide model of a coal seam. Often it can be assumed that $\mu_r = u_f$ and $\rho_r = \rho_f$. (Here, μ is the rigidity and ρ is the density). For SH waves with velocity V_{SH}, the Lamé relationships yield

$$V_{SH} = (\mu_r/\rho_r)^{\frac{1}{2}}$$

In the lower diagram, the condition for constructive interference is $\ell(\theta) = n\lambda$ where n is an integer and λ is the wavelength of the wave forming the wave front.

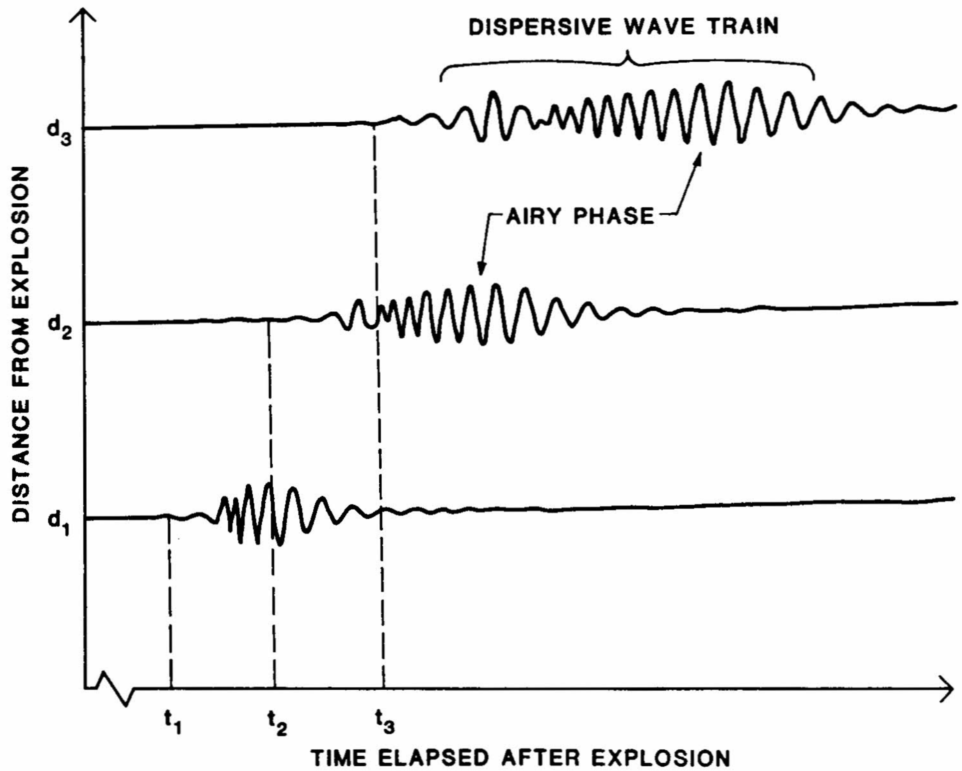


Figure 2. Seismograms of transmitted waves showing distances to geophones in bore holes d_1, d_2, d_3,..., times of first arrivals of the detonation pulse of the explosion t_1, t_2, t_3,.. , and the Airy phase.

A detailed analysis of the dispersive characteristics of the wave trains which make up the seismogram can be used to determine information about the properties of the seam through which the waves have travelled. The point of common contact between the indirect measurements contained in the seismograms and the characteristics of the seam is the group velocity curve (as a function of frequency) which characterizes the dispersive properties of the medium through which the waves have passed (see Figure 3).

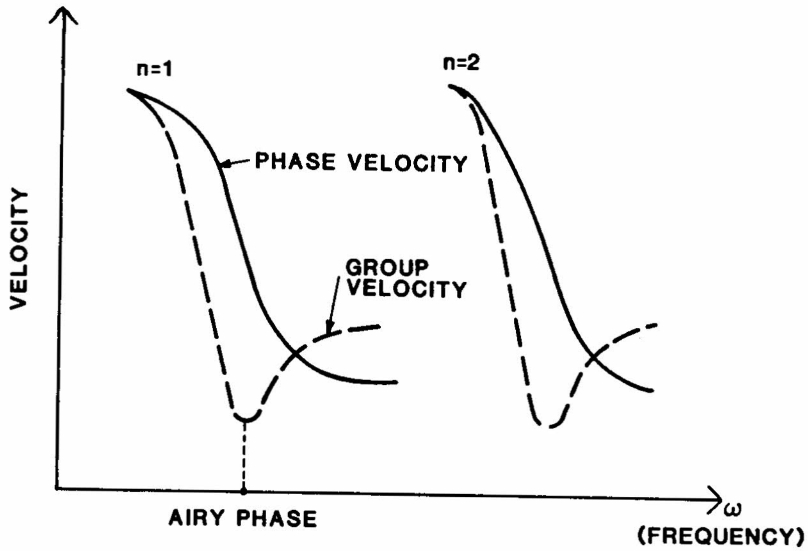


Figure 3. The phase group velocity curves represent the point of common contact between the rectilinear wave guide model of the coal seam and the information contained in the seismograms of the transmitted SH waves. The phase velocity V_p can be determined in terms of the parameters which define the rectilinear model, while estimates of the group velocity V_g can be obtained from an analysis of the dispersive structure of the seismograms. The phase and group velocities thereby obtained can be related to each other through the standard formulas for these velocities, viz.

$$V_g = d\omega/dk,\ V_p = \omega/k,$$

where k and ω denote the wave number and frequency of the particular wave under examination.

On the one hand, a mathematical analysis of the propagation characteristics of wave guides can be used to relate the elastic and geometric parameters of the seam to the phase velocity of the waves generated by constructive interference (see Krey (1963), Buchanan (1978)). These waves correspond to the standing wave (separation of variable) solution of the wave equation which models the wave propagation in the seam. The standard definitions for phase and group velocities can then be used to relate these parameters to the group velocity curve for the elastic waves which propagate in the seam.

On the other hand, since the seismograms only see the elastic waves which can propagate in the seam (due to constructive interference), data analysis techniques can be applied to them to extract information about their group velocity as a function of frequency (see Dziewonski et al. (1969), Dziewonski & Hales (1972)). The quality of this information will depend heavily on the data rate used to record the seismograms, and the number and distribution of the geophones.

Thus, the second problem posed by ACIRL has a two-fold character:

i. Identify specific features of the group velocity curve which constrain particular characteristics in the model of the coal seam being used.

ii. Analyse the standard and develop new techniques for extracting information about specific features of the group velocity curve from the seismogram.

To illustrate, consider the velocity and frequency of the Airy phase, which corresponds to the wave packet defined by the minimum on a group velocity curve. With respect to (i), the Airy phase information is known to constrain seam thickness. For example, it is known that the wavelength of the Airy phase of the fundamental Love mode is always greater than the seam thickness for all velocity contrasts, while the Airy wavelengths of the 3rd and higher modes are less than the seam thickness, (see Greenhalgh & King, 1981).

On a seismogram, the Airy phase (normally) corresponds to the maximum amplitude component in the wave train. It is the only component in the wave train for which the amplitude does not decrease due to dispersion. Thus, with respect to (ii), the velocity and frequency of the Airy phase is easily estimated from the seismograms. For example, the velocity can be estimated using the same technique described above for determining the SH wave velocity in the roof and floor of the seam. The frequency can be obtained from a

Fourier analysis of the Airy phase component in the seismograms.

3.3 Summary of MISG deliberations

Because it represents the simplest situation without hiding the essential nature of the problem, attention concentrated on the analysis of SH seismic waves.

For the seam model shown in Figure 1, the group velocity V_g (as a function of the frequency ω) of the standing wave solutions of the wave equation for this configuration was shown to have the form (symbols defined in Figure 1)

$$V_g = \{\lambda_r \lambda_s^2 \mu_s kd \sec^2(\lambda_s d) + k\mu_r(\lambda_s^2 + \lambda_r^2)\}/$$

$$\{\omega\mu_r c_s \lambda_s^2 + c_r^2 \lambda_r^2)/c_r^2 c_s^2 + \lambda_r \lambda_s^2 \mu_s \sec^2(\lambda_s d)/c_s^2\}$$

$$\lambda_s^2 = \omega^2/c_s^2 - k^2, \ \lambda_r^2 = k^2 - \omega^2/c_r^2,$$

$$c_s^2 = \mu_s/\rho_s, \ c_r^2 = \mu_r/\rho_r.$$

It illustrates clearly the difficulty which the solution of (i) poses. The parameters of the seam determine V_g as a function of ω in a highly non-linear way, and thereby yield a situation where there is, in general, no obvious way in which a particular feature of the seam has a simple interpretation in terms of the graphical structure of V_g as a function of ω.

Because it is not difficult to propose and examine different ways in which information about the group velocity as a function of frequency can be extracted from the seismograms of the transmitted waves, the study group's examination of (ii) proved quite productive. If nothing else, it clarified a number of points about the numerical analysis of the different methodologies in current use. Discussions ranged over a variety of topics including

i. An examination of the advantages and disadvantages of the standard methods in current use. For example, if the seismograms had been recorded at a large number of boreholes all close together, then standard Fourier analysis methods could be used to recover the desired information. Since this is not possible, the data analysis techniques which are used aim to exploit the fact that, because of the high data rates used in recording the seismograms, the dispersive structure of the wave train defining a seismogram is recorded in great detail and reasonable accuracy.

ii. A review of Fourier (spectral) methods for the analysis of the dispersive structure in a wave train.

iii. An examination of direct averaging of the Airy phase estimates computed directly from each of the seismograms.

iv. Radon transform enhancement of phase front information (Ray-Chaudhury et al., 1985). This technique is particularly suited to situations where the phase fronts lie more or less along straight lines. The discrete Radon transform of the ensemble of seismograms is filtered using the semblance of the ensemble and the back transformed using the known discrete inverse Radon transform to yield the enhanced wave fronts. This methodology clearly represents a seismic counterpart of the Wiener filtering of time series.

Other aspects of the problem were examined including resonance induced transmission in seams of the same thickness displaced by faults.

3.4 Further work

Though no significant progress was made with (i) at the MISG, there was general agreement that it represented such an important aspect of the problem that it should be pursued because of the obvious advantages which would result if specific features of the group velocity curve could be related more directly to characteristics in a model of a coal seam.

There was complete agreement that there was considerable scope for refining and supplementing present methodologies for extracting estimates of specific structure of the group velocity curve from the seismograms. In particular, strategies for pooling the information from transmission and reflection seismology and for SH and P-SV seismic waves needed to be developed in order to enhance the accuracy of predictions made about seam structure.

Finally, it was clear that a better understanding of the problem depended heavily on the need to analyse the standing wave solutions of the wave equation

$$\nabla^2 u = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}, \qquad c = \text{velocity},$$

in non-rectilinear seams.

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