Principal Moments of Inertia of a Vehicle Engine Assembly

Engines are attached to vehicle bodies by flexible mounts, traditionally of rubber. One purpose of these is to isolate the body and consequently the passengers from most vibrations generated by the engine and the driveline. The rear engine mount is located under the gearbox area near a node of oscillation of the engine and driveline assembly. Vibration transmission to the two front engine mountings (located symmetrically on opposite sides of the engine) is minimized if they are located at points beside a single point determined by the *torque roll axis* (figure 1) and *centre of percussion* corresponding to the rear engine mounting. This point can be computed from a knowledge of the centre of mass and inertia tensor of the engine assembly.

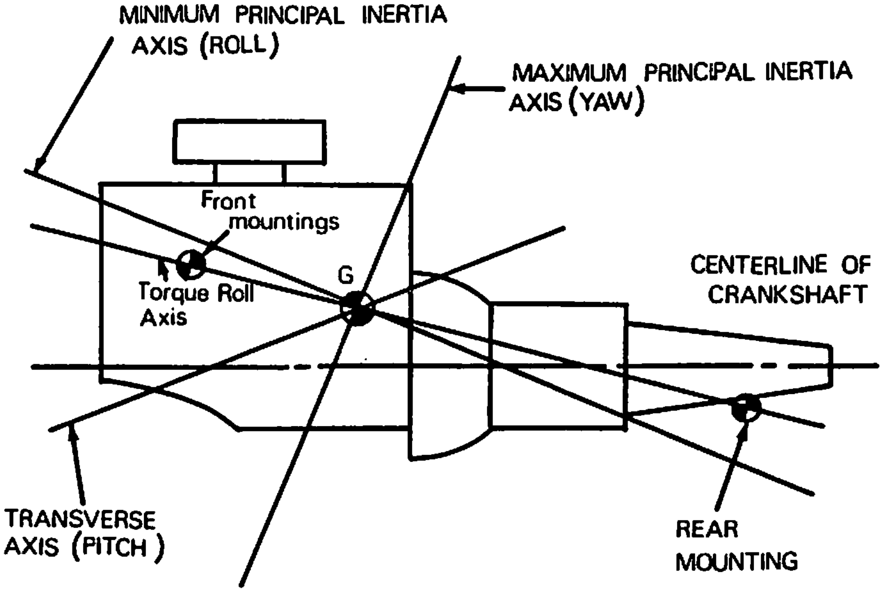

Figure 1: Engine and Driveline Assembly.

The centre of mass is easily determined by static balancing. Ford at Geelong currently measures the inertia tensor using a trifilar torsional pendulum. This requires continual repositioning of the engine assembly in a frame and takes two to three men up to three days, while giving errors of 10% at best. Ford asked for an improved method of measurement and we explored methods which involve more subtle aspects of rigid body dynamics.

A literature search revealed other devices, such as modified torsional pendulums and frames rotating on shafts, but all were reported to be defective in one or more ways such as being inaccurate, time consuming or costly. Ford hoped to make a device using hydraulic jacks which it currently employs in full vehicle rigs. Each jack is equipped with accurate gauges for measuring force ($\approx 0.25\%$ error) and displacement ($\approx 1\%$ error). Accelerometers ($\approx 0.5\%$ error) are also available. The jacks are activated by computer controlled servomechanisms, so that any desired waveform can be given to the jack. Ford has much experience with this technology and with analysis of vehicle responses, so it is very desirable to utilize this expertise.

We identified several basic requirements of a jack activated rig (JAR).

(a) The engine must be rotated about several axes: three perpendicular axes is sufficient to identify all six elements of the inertia tensor if all components of torque are measured for each of the three axes.

(b) Linear motion of the centre of mass G should not be large compared to rotational motion about G. To see this, note that the period

$$2\pi[(h^2 + k^2)/hg]^{\frac{1}{2}}$$

of a compound pendulum gives a poor measure of the radius of gyration k about G if G is a relatively large distance h from the pivot 0.

(c) Engine mounting on the JAR should be a simple, once-only operation.

The greatest freedom of motion is achieved by a JAR consisting of a frame attached by pivots to the tops of three tripods, each tripod comprising 3 jacks. This is capable of producing rotations (of less than 2π) about any axis, in particular about any fixed axis through G. The resulting equations for the inertia tensor involve the nine jack extensions, their nine extension rates, their nine extension accelerations and their nine forces, together with three constraints arising from the rigidity of the frame. The equations involve a time dependent transformation from jack lengths to useable laboratory coordinates and from these to moving coordinates attached to the frame. This general JAR was rejected on several grounds including:

(a) the large number of variables involved and their sources of error,

(b) the need to develop a complex computer program to operate it

(c) the excessive cost of nine jacks and their peripherals.

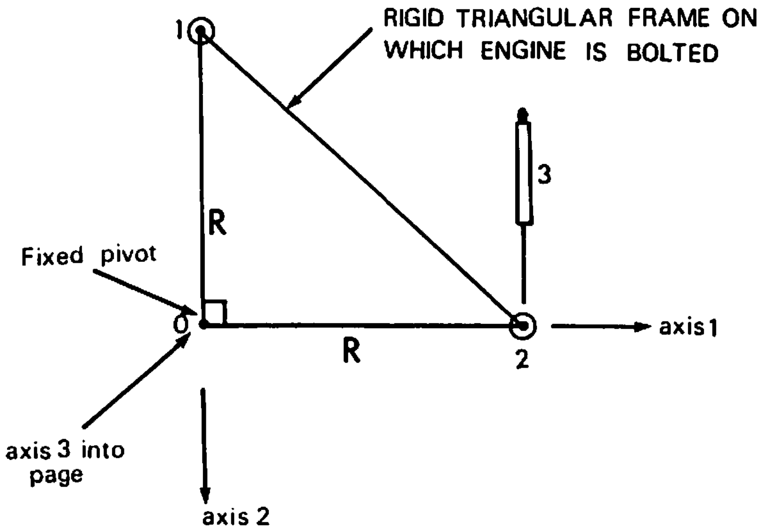

Figure 2: Plane view of rig. Jacks 1 and 2 are aligned vertically below the triangular frame. Jack 3 is in the horizontal plane.

The simple JAR illustrated in Figure 2 was recommended. It comprised a rigid horizontal frame in the shape of a right angled isosceles triangle, attached to a fixed pivot at 0, to vertical jacks 1 and 2 as shown and to a horizontal jack at 2. Locating G directly above 0 or as close as possible would minimize the linear motion of G which was judged to be, at worst, comparable to the angular motion (i.e. $h \approx k$). Keeping jacks 2 and 3 fixed in length and activating jack 1 results in rotation about the fixed axis 1. The equations for rate of change of angular momentum take the form

$$\begin{aligned} \Gamma_1 &= I_{11}\dot{\omega} \\ \Gamma_2 &= I_{12}\dot{\omega} - I_{31}\omega^2 \\ \Gamma_3 &= I_{31}\dot{\omega} + I_{12}\omega^2 \end{aligned} \qquad (1)$$

where ω is the angular velocity (about the axis 1), the Γ_i are the components of torque about 0 along the axes fixed to the frame and the I_{ij} are the components of the inertia tensor relative to the same origin and axes. The I_{ii} are moments of inertia and the $-I_{ij}$ $(i \neq j)$ are products of inertia. With the frame of side $R = \frac{1}{2}m$ and a jack displacement of ±50 mm, the frame will deviate from the horizontal by a maximum angle of $\theta \approx 0.1$ radians. The error in the approximations $\sin\theta \approx \theta$ and $\cos\theta \approx 1$ are therefore of order $\frac{1}{2}\theta^2$ or $\frac{1}{2}$%. Thus to this approximation we have $\omega = l_1/R$, $\dot{\omega} = \ddot{l}_1/R$ where l_1 is the instantaneous length of jack 1 and R is shown in Figure 2. If G is directly above 0, then to this accuracy $\Gamma_i = F_i/R$ where F_i is the force exerted by jack i on the rig. Otherwise the engine weight contributes to Γ_1 and Γ_2.

Equations (1) can be solved for I_{11}, I_{12} and I_{13} for any choice of time varying input data ω. If measurement errors are considered important a linear regression is appropriate.

If the engine is fitted to the rig so that two of its principal axes at 0 are in the plane of the rig (i.e. of axes 1 and 2) then $I_{13} = I_{23} = 0$. Then equations (1) give one estimates $\Gamma_1/\dot{\omega}$ and Γ_3/ω^2 for I_{12}. A simple time averaging of these may be adequate in view of the small errors involved. Two principal axes are often assumed to lie in the plane containing the crankshaft and the centre of the cylinder block. With this assumption, the above simplifications may be adequate. For greater accuracy the full equations (1) should be employed.

Simple sinusoidal displacement of the jacks, or narrow band white noise are suitable inputs. Low frequencies are useless since they yield small force measurements which may be comparable to measurement errors. High frequencies are to be avoided since they may excite resonances (typically above 50 Hertz) and bending of frame or engine attachments resulting in phase shifts associated with elastic waves. An initial input of white noise could be used to spot these effects: subsequently they could be avoided.

Keeping jacks 1 and 3 fixed in length and activating jack 2 results in rotation about an axis which deviates slightly from axis 2. Ignoring this deviation gives an error of about $\frac{1}{2}$% again. Equations of the form (1) for I_{22}, I_{23} and I_{12} are obtained. Finally activating jack 3 alone gives equations for I_{33}, I_{13} and I_{23}. For a proper error analysis the three sets of equations should be pooled.

The inertia tensor of the frame alone can be measured by the same method. Subtracting this from the above gives the inertia tensor of the engine assembly alone about 0. The parallel axes theorems give the inertia tensor about G. Solving the eigenvalue problem then gives the principal axes and moments of inertia.

REFERENCES

Atkin, R.H., *Classical Dynamics* Wiley, New York (1959).

McCuskey, S.W., *An Introduction to Advanced Dynamics* Addison-Wesley, Massachusetts (1959).

Harris, C.M. and Crede, C.E., *Shock and Vibration Handbook*, Vol. 1, McGraw-Hill, New York (1951) pp.34-38.

FUS. Test Procedures - AMTM L-73-21 and 23, Ford Motor Company "Test procedure: mass moments of inertia of engine and/or transmission assembly".

Ford NAAO FWD Powertrain Mounts Design Guide, ed. R.R. Fullerton, Ford Motor Company (1984). Appendix 4 "Determination of torque roll axis location".

Crede, C.E., *Vibration and Shock Isolation*, Wiley, New York (1951) pp.38, 44-58, 73-87, 216-242.

Rover, W.G. and Petaga, A.E., *Design Guide: Engine Mounts*, Ford Motor Company (1967).

Bevan, T., *The Theory of Machines*, Longmans (1967) pp.38, 39.