ROLLING MILL PERFORMANCE

A rolling mill may be modelled as a number of inertial masses coupled by torsional springs. The question considered by the Study Group was whether the parameters of the system could be determined from measurements of the torques and accelerations at a number of points in the system. A number of related aspects such as resonances, torque amplification factors and parameter identification were also examined. The group concluded that frequency analysis and lumped mass models were potentially useful for the analysis of rolling mill performance.

1. Introduction

A drive system for hot rolling mills can be regarded as an assemblage of rotating inertial masses connected by torsional springs, subject to torsional forces. The peak values of the torque in the system may be several times the steady state or mean value and, as a consequence, it is possible the high torque values might severely stress and damage mill components including the drive motors.

When the inertial masses (moments of inertia) and torsional spring constants are known, it is possible to simulate the mill performance. This is essentially equivalent to calculating the effective "mill transfer function" and such calculations are performed routinely by Industrial Systems Pty Ltd to characterise the attainable level of performance such as speed response (subject to constraints on maximum torque on key components to avoid mechanical problems). In practice however the mill parameters are not always readily available and the question posed by Industrial Systems is whether these parameters can be deduced from measurements of instantaneous torque and acceleration at a number of points in the system.

2. Mathematical model

The rolling mill was modelled as a system of n inertial masses coupled by $n-1$ torsional springs as shown in figure 1. In addition, linear damping proportional to the angular velocity was assumed. This leads to the system of differential equations

$$M\,\mathbf{x}'' + B\,\mathbf{x}' + D^T K D \mathbf{x} = \mathbf{f} \tag{1}$$

where

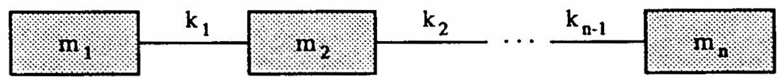

Figure 1: Schematic Representation of Rolling Mill

$$
\begin{aligned}
\mathbf{x} &= (x_1, x_2, \dots, x_n)^T \\
\mathbf{x}' &= \frac{d\mathbf{x}}{dt} \\
\mathbf{x}'' &= \frac{d^2\mathbf{x}}{dt^2} \\
f &= (f_1, f_2, \dots, f_n)^T \\
M &= \mathrm{diag}(m_1, m_2, \dots, m_n) \\
B &= \mathrm{diag}(b_1, b_2, \dots, b_n) \\
K &= \mathrm{diag}(k_1, k_2, \dots, k_{n-1})
\end{aligned}
$$

$$
D = \begin{bmatrix} 1 & -1 & 0 & 0 & . & . & . & 0 & 0 \\ 0 & 1 & -1 & 0 & . & . & . & 0 & 0 \\ . & . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & & & & 1 & -1 \end{bmatrix}
$$

Typical values (in SI units) for m_i and k_i are given in table 1. Estimates for the damping parameters b_i were not available. Most investigations of the problem required solution of (1), particularly for special cases. Some of these solutions are summarised below.

Component	Moment of Inertia	Torsional Stiffness
1 Motor	24078	1.64×10^{8}
2 Motor	23911	4.28×10^{7}
3 Coupling	1450	2.82×10^{7}
4 Gears	5665	2.05×10^{6}
5 Spindle	680	1.74×10^{6}
6 Pinions	133	8.84×10^{5}
7 Rolls	417	

Table 1: Typical values for parameters (in SI units)

2.1 Steady state: f = constant

When **f** constant, the steady state solution of (1) is

$$\mathbf{x}' = \Theta' \mathbf{e}, \quad \mathbf{e} = (1, 1, \dots, 1)^T$$

where the angular velocity Θ' can be calculated from the linear equations

$$\left[B\mathbf{e} | D^T K\right] \left[\frac{\dot{\Theta}}{D\mathbf{x}}\right] = \mathbf{f}$$

The above equation also provides the solution of the relative rotation $D\mathbf{x}$.

2.2 Non-damped system: $B = 0$

(a) *Resonance*

Let $0 = \omega_0 < \omega_1 < \dots < \omega_{n-1}$ satisfy the generalized eigenvalue problems

$$\omega_k^2 M \mathbf{x}_k = D^T K D \mathbf{x}_k, k = 0, \dots, n-1$$

For $k = 0$, $\omega_0 = 0$ and $\mathbf{x}_0 = \mathbf{e}$ which corresponds to a constant angular velocity in the absence of a forcing term. For $k > 0$, ω_k are the resonant frequencies since a forcing term of these frequencies will result in a solution that increases in amplitude with time. Specifically,

$$\begin{aligned} M\mathbf{x}'' + D^T K D\mathbf{x} &= 2\omega_k M \mathbf{x}_k \cos(\omega_k t) \\ \mathbf{x}(0) &= \mathbf{0} \\ \mathbf{x}'(0) &= \mathbf{0} \end{aligned}$$

has the solution

$$\mathbf{x} = t\sin(\omega_k t)\mathbf{x}_k$$

(b) **f** *is constant*

When **f** is constant, the solution of

$$\begin{aligned} M\ddot{\mathbf{x}} + D^T K D\mathbf{x} &= \mathbf{f} \\ \mathbf{x}(0) &= \mathbf{0} \\ \mathbf{x}'(0) &= \mathbf{0} \end{aligned}$$

is

$$\mathbf{x} = A^{-1}(I - \cos(A^{1/2}t))M^{-1}\mathbf{f} \tag{2}$$

where

$$A = M^{-1}D^T K D$$

and

$$A^{-1}(I - \cos(A^{1/2}t)) = \sum_{k=0}^{\infty} \frac{(-1)^k A^k t^{k+2}}{(2k+2)!}$$

2.3 Periodic solution (non-resonant)

When **f** is periodic with period T, then

$$\mathbf{f}(t) = \sum_{k=-\infty}^{+\infty} \alpha_k \exp(\frac{2\pi i k t}{T})$$

and the periodic solution of (1) is given by

$$\mathbf{x}(t) = \sum_{k=-\infty}^{\infty} \beta_k \exp(\frac{2\pi i k t}{T})$$

where

$$\beta_k = \left[D^T K D + \frac{2\pi i k}{T}B - (\frac{2\pi k}{T})^2 M\right]^{-1} \alpha_k$$

2.4 General solution

The general solution of (1) can be calculated by inverting its Laplace transform. Specifically, the Laplace transform is

$$X(p) = \left[p^2 M + pB + D^T K D\right]^{-1} \left\{p\mathbf{x}(0) + \mathbf{x}'(0) + \mathbf{F}(p)\right\} \tag{3}$$

3. Torque amplification factor

As discussed previously, an important aspect of hot mill operation is the maximum torque on the shafts. If the damping coefficients are small, and a constant load **f** is applied to a system at rest then, from the previous section, we have

$$\mathbf{x} = A^{-1}(I - \cos(A^{1/2}t))M^{-1}\mathbf{f}.$$

Hence, the torques on the shafts are given by

$$\begin{aligned} \mathbf{T} &= -KD\mathbf{x} \\ &= -KDA^{-1}(I - \cos(A^{1/2}t))M^{-1}\mathbf{f}. \end{aligned}$$

For hot rolling, we are primarily interested in the load applied via the rolls - that is, $\mathbf{f} = f\mathbf{e}_n$. The torque amplification factors are defined to be the ratios of the maximum torque on the shafts to the applied load. Thus

$$\eta_j = \sup_t |\mathbf{e}_j^T KDA^{-1}(I - \cos(A^{1/2}t))M^{-1}\mathbf{e}_n|$$

For $n = 2$, it is easy to show that

$$\eta_1 = \sup_t \left\{ \frac{m_1(1 - \cos\omega_1 t)}{(m_1 + m_2)} \right\}$$

where

$$\omega_1 = \left\{ \frac{k_1(m_1 + m_2)}{m_1 m_2} \right\}^{1/2}$$

Thus,

$$\eta_1 = \frac{2m_1}{(m_1 + m_2)} < 2$$

and, as a consequence, the maximum torque in a system with two inertial masses can be up to twice as large as the applied torque. For $n = 3$, the situation seems to be much worse. It can be shown (see *e.g.* Church, 1948) that

$$\eta_1 = \frac{2m_1\omega_2^2}{(m_1 + m_2 + m_3)(\omega_2^2 - \omega_1^2)}$$

which appears to become unbounded as $\omega_2 \to \omega_1$. Similarly for $n \geq 3$ we can obtain expressions for η by inversion of the Laplace transform (3) (with $B = 0$) whose numerators contain products of the form $\prod_{j \neq i}(\omega_i^2 - \omega_j^2)$. These products arise as residues of $1/\det(p^2M + D^TKD)$ when Cramer's rule is used to provide a representation of $[p^2M + D^TKD]^{-1}\mathbf{e}_n$.

The question of whether torque amplification factors will become unbounded when the resonant frequencies approach each other is clearly crucial in hot mill design. If this is the case, it means that the resonant frequencies need to be well separated. The group spent a substantial amount of time on this question and reached the following conclusions:

- no example was found for which $\eta_i \to \infty$
- for $n = 3$ it is possible to construct an example ($k_1 = k_2, m_1 = m_3, m_2 \to \infty$) for which the torque amplification factor remains bounded even though $\omega_1 \to \omega_2$
- for $n \geq 3$ it follows from (2) that the torque amplification factor is bounded if $m_1/m_i, i = 1, \dots, n$ are bounded
- no consensus could be reached on the question of whether an unbounded torque amplification factor is possible.

4. Parameter identification

Although it is possible, in principle, to apply loads to each component and measure their angular velocity, it is practical to apply only a known torque at the motor and measure its angular velocity. Intuitively, it is preferable to apply a load that is 'rich' in high frequencies and two loads of this type were considered.

Impulsive load

If the system is at rest and an impulsive load $\mathbf{f} = f\delta(t)\mathbf{e}_1$ is applied, then the Laplace transform of x_1 is given by

$$X_1(p) = \mathbf{e}_1^T f \left[p^2 M + pB + D^T K D\right]^{-1} \mathbf{e}_1 \tag{4}$$

Now for large p we can write

$$X_1(p) \approx \sum_{k\geq 1} \frac{x_1^{(k)}(0)}{p^{k+1}}$$

In principle, we know $x_1^{(k)}(0)$ and we can expand the right hand side of (4) in powers of p^{-1} and equate coefficients to obtain a system of nonlinear equations for the unknown parameters. For example

$$x_1'(0) = f/m_1$$

allows us to calculate m_1. However the method requires the calculation of high order derivatives at $t = 0$ and is clearly not practical for identifying more than the first few parameters. Furthermore, an impulse is not a feasible input although the analysis is easily modified for more general applied loads.

Periodic load

As discussed previously, a periodic load of the form

$$\mathbf{f}(t) = \sum \mathbf{e}_1 \alpha_k \exp(\frac{2\pi ikt}{T})$$

will lead to a periodic solution of the form

$$x_1(t) = \sum \beta_k \exp(\frac{2\pi ikt}{T}) \tag{5}$$

where

$$\beta_k = \mathbf{e}_1 \left[D^T KD + i(\frac{2\pi k}{T})B - (\frac{2\pi k}{T})^2 M \right]^{-1} \mathbf{e}_1 \alpha_k$$

If $\mathbf{f}(t)$ and $x_1(t)$ are known then, in principle at least, α_k and β_k can be calculated. Thus, (5) represents an overdetermined non-linear system of equations for K, B and M. An attempt to solve this system in a least squares sense during the Study Group was considered to be impractical.

Generally speaking, the problem of parameter identification was thought to be difficult in practice due to the ill-posed nature of the problem. Filtering techniques (Kalman filtering in particular) were considered but not pursued due to time constraints at the Study Group and a lack of expertise in this area.

5. Resonance

Since an important feature of the mechanical system considered are the resonant frequencies, the group spent some time on the determination of these frequencies. Two approaches were pursued.

Frequency analysis

From Section 2, the periodic solution of (1) is given by

$$\mathbf{x}(t) = \sum_{k=-\infty}^{+\infty} \beta_k \exp\left(\frac{2\pi ikt}{T}\right)$$

where

$$\beta_k = \left[D^T KD + \frac{2\pi ik}{T} B - (\frac{2\pi k}{T})^2 M \right]^{-1} \alpha_k$$

When B is small, we expect that $|\beta_k|$ will be large when

$$\frac{2\pi ik}{T} \sim \omega_j, j = 1, 2, \ldots, n-1$$

Thus, if we plot $|\beta_k|^2$ against k we should in principle be able to identify the approximate locations of the resonant frequencies. This idea was simulated during the Study Group by Industrial Systems Pty Ltd using a ramp input for the torque on the motor and a measurement of the angular velocity of the motor. That is,

$$\mathbf{f}(t) = f(t)\mathbf{e}_1$$

is given and $\mathbf{e}_1^T \dot{\mathbf{x}}$ is 'measured'. The quantity $|k\beta_k^T \mathbf{e}_1|^2$ was plotted against k and it was possible to determine the first three or four resonant freqencies. Of course, a simulation does not address the question of noise and as no information was available on measurement error this was not pursued. The group noted that frequency analysis was a well established subject (see e.g. Randall, 1987). Industrial Systems Pty Ltd felt that this might be a useful tool for them and they should analyse some real data with a view to determining the low frequency resonances.

Lumped mass simplifications

The model given by (1) is a 'lumped mass' model since each of the inertial masses will consist of a number of components. It is therefore natural to ask whether further lumping of the inertial masses is possible to simplify the problem. Table 2 shows some of the simplifications considered while table 3 shows the corresponding resonant frequencies (in Hertz).

Model	Masses Lumped						
1	1	2	3	4	5	6	7
2	1,2,3	4	5,6	7			
3	1,2,3	4,5	6,7				
4	1,2,3	4,5,6	7				
5	1	2					

Table 2: Lumped mass models

Model	Resonant Frequencies					
1	4.7	8.6	10.9	18.8	23.8	36.4
2	4.42	8.50	9.78			
3	4.52	8.46				
4	5.17	8.4				
5	18.6					

Table 3: Frequencies for lumped mass models

It is clear that the lower frequencies can be determined quite well with the lumped mass simplification. In addition specific frequencies can in some instances be associated with specific components. For example the resonance of 18.8 can clearly be associated with the resonance of the motors in isolation.

6. Conclusions

The following conclusions were reached by the group:

- Further work is required to ascertain if identification of all parameters in a rolling mill is practical.
- A better understanding of torque amplification factors are desirable and the fact that two resonant frequencies are close does not necessarily mean that the amplification factor is large.
- Frequency analysis is potentially a very useful tool in mill design and its use should be investigated.
- Lumped mass models may be useful in model simplification and thus provide useful insight.
- Non-linear effects due to drive converters are potentially important.
- Further work is required to assess the effects of a feedback loop.

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