A SIMPLE MODEL FOR THE COLD ROLLING OF METAL FOIL

The flow of perfectly plastic material between elastic rolls is examined as a model for rolling of foils. Key non-dimensional parameters are identified and the potential for approximate solutions, based on an asymptotic analysis, is examined.

1. Introduction

Industrial Automation Services Pty Ltd (IAS) provides technical consulting and software engineering services to the steel, aluminium and coal industries. They are currently implementing computer models for cold rolling of foil based on mathematical models that have recently been published in the literature (Fleck and Johnson, 1987, and Fleck *et al.*, 1992).

A key feature of foil rolling is the fact that the elastic deformation of the work roll is strongly coupled to the plastic flow in the nip (the region between a and b in figure 1). This is also the case in elasto-hydrodynamic lubrication (see for example Johnson, 1984) where there is fluid flow in the nip. However, in this case a number of simple and useful approximate and asymptotic solutions have been derived. Since the qualitative nature of the solutions with plastic and fluid flows have been observed by IAS to be similar, the question posed by IAS to the study group was whether asymptotic and approximate solutions could also be derived for foil rolling.

2. Notation

R	—	radius of work roll
$h = h(x)$	—	half width of strip in roll gap at x
h_i	—	half width of strip at inlet
h_o	—	half width of strip at exit
$v = v(x)$	—	velocity of strip at x
V	—	peripheral velocity of work rolls
$\mu = \mu(x)$	—	coefficient of friction at x
μ_0	—	coefficient of friction
$\sigma_{xx} = \sigma_{xx}(x)$	—	normal stress on yz plane at x
$\sigma_{yy} = \sigma_{yy}(x)$	—	normal stress on xz plane at x
$p = p(x) = -\sigma_{yy}(x)$	—	pressure at x
$S = S(x)$	—	frictional shearing stress at x
k	—	yield stress of strip

E	—	Young's modulus of work roll
ν	—	Poisson ratio of work roll
K	—	Elastic constant in mattress approximation
$e = e(x)$	—	elastic deformation of work roll at x
a	—	x co-ordinate of inlet
b	—	x co-ordinate of outlet
c	—	x co-ordinate of end of first deformation region
d	—	x co-ordinate of beginning of second deformation region
T_i	—	Tension on strip at inlet
T_o	—	Tension on strip at outlet
p_i	—	Pressure on strip at inlet
p_o	—	Pressure on strip at outlet
G_1	=	$\sqrt{R/h_i}\,\mu_0$
G_2	=	kKR/Eh_i
G_3	=	$(1-\nu^2)kK\sqrt{R}/E\sqrt{h_i}$

3. Derivation of the model

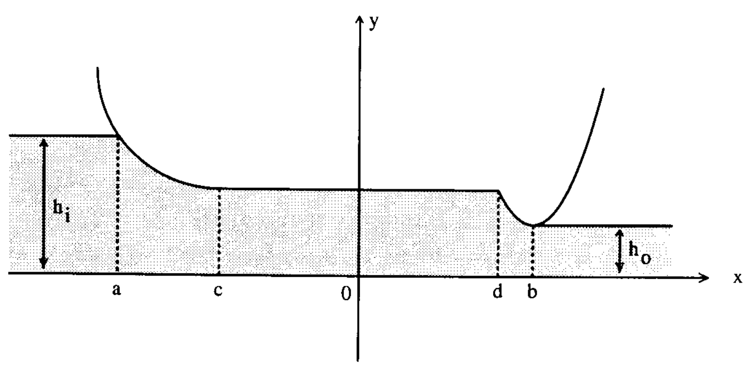

Figure 1: Schematic of foil rolling.

If we assume that stresses do not vary through the strip and that the gap (the region between c and d in figure 1) is essentially parallel to the x axis, the equation of equilibrium in the x direction is given by

$$\frac{d}{dx}(h\sigma_{xx}) - \frac{dh}{dx}\sigma_{yy} + S = 0 \tag{3.1}$$

Furthermore if σ_{xx} and σ_{yy} correspond to the principal stresses, the yield condition for ideal plastic flow is

$$\sigma_{xx} - \sigma_{yy} = k \tag{3.2}$$

On assuming that the strip is in contact with the roll over the entire nip region and that the frictional shearing stress is given via Coulomb friction, we obtain

$$S = \mu \sigma_{yy} \tag{3.3}$$

where

$$\mu(x) = \begin{cases} \mu_0 & \text{if } v - V > 0 \\ 0 & \text{if } v - V = 0 \\ -\mu_0 & \text{if } v - V < 0 \end{cases} \tag{3.4}$$

Thus on combining (3.1), (3.2) and (3.3), we obtain the plastic flow equation

$$h\frac{dp}{dx} - k\frac{dh}{dx} + \mu p = 0 \tag{3.5}$$

The half width of the strip is given by the differential equation

$$\frac{dh}{dx} = \frac{x}{R} + \frac{de}{dx} \tag{3.6}$$

and during the MISG, we considered the following two approximate relationships for the elastic deformations

$$e = \frac{KRp}{E} \tag{3.7a}$$

$$\frac{de}{dx} = \frac{-2(1-\nu^2)}{\pi E} \int \frac{p(s)}{x-s} ds \tag{3.7b}$$

Equation (3.7a) is the mattress approximation which simply says that the elastic deformation is proportional to the applied pressure. While it is known to be a poor approximation, it is quite useful as it can often lead to insight into elastic contact problems. A more appropriate (and also more complicated) relation for the deformation is given by (3.7b) which is derived from Hertz contact theory when friction is neglected. Since $\mu_0 << 1$, this is a reasonable approximation. It should be noted that the integral in (3.7b) must be interpreted as a Cauchy principal value integral.

The plastic flow equation (3.5) and the elastic deformation equation (either (3.7a) or (3.7b)) need to be augmented by some boundary conditions, parameters and constraints. They are

$$h(a) = h_i \tag{3.8a}$$

$$h(b) = h_0 \tag{3.8b}$$

$$p(a) = p_i = k - \frac{T_i}{h_i} \tag{3.8c}$$

$$p(b) = p_0 = k - \frac{T_0}{h_0} \tag{3.8d}$$

$$p(x) = 0, \quad x > b \tag{3.8e}$$
$$p(x) = 0, \quad x < a \tag{3.8f}$$
$$\frac{dh}{dx} < 0 \tag{3.8g}$$

As is indicated in figure 1, there are essentially three regions in the mill nip. At the inlet there is an initial region of reduction $a < x < c$. This is followed by a central region $c < x < d$ where little or no reduction occurs. Finally, there is another region of reduction $d < x < b$ at the outlet. In Fleck and Johnson (1987) and Fleck *et al.* (1992) these regions are part of the model formulation and the plastic flow equation (3.5) is replaced by

$$p(x) = 0, \quad x < a \tag{3.9a}$$
$$h\frac{dp}{dx} - k\frac{dh}{dx} - \mu_0 p = 0, \quad a < x < c \tag{3.9b}$$
$$\frac{dh}{dx} = 0, \quad c \leq x < d \tag{3.9c}$$
$$h\frac{dp}{dx} - k\frac{dh}{dx} + \mu_0 p = 0, \quad d < x < b \tag{3.9d}$$
$$p(x) = 0, \quad x > b \tag{3.9e}$$

4. MISG discussion

A first step in examining foil rolling was to cast the equations in non-dimensional form with a view to simplifying the equations and to determine important non-dimensional groups. The following scaling was used

$$\begin{aligned}
p &\to kp\\
x &\to \sqrt{Rh_i}\,x\\
a &\to \sqrt{Rh_i}\,a\\
b &\to \sqrt{Rh_i}\,b\\
c &\to \sqrt{Rh_i}\,c\\
d &\to \sqrt{Rh_i}\,d\\
e &\to h_i e\\
h &\to h_i h
\end{aligned}$$

As a consequence, we obtain the following scaled equations

$$\frac{dh}{dx} = x + \frac{de}{dx} \tag{4.1}$$

$$h\frac{dp}{dx} - \frac{dh}{dx} + \hat{\mu}p = 0 \tag{4.2}$$

and

$$e = G_2 p \tag{4.3a}$$

$$\frac{de}{dx} = \frac{-2G_3}{\pi} \int \frac{p(s)}{x-s} ds \tag{4.3b}$$

where

$$\hat{\mu}(x) = \begin{cases} G_1 & \text{if } v - V > 0 \\ 0 & \text{if } v - V = 0 \\ -G_1 & \text{if } v - V < 0 \end{cases} \tag{4.4}$$

Typical values for the parameters are given in table 1. From these values we find that the length scale $\sqrt{Rh_i}$ is consistent with the solutions calculated using the IAS model even though this length is based on scaling of a rigid work roll. Of the three non-dimensional groups G_1, G_2 and G_3, only G_2 potentially exhibits an extreme behaviour (ie: is either large or small) if we assume that $K = O(1)$. This is of concern as it suggests an inconsistency between the two approximations for elastic deformation of the work roll. Since G_2 is the parameter associated with the mattress approximation we expected it to be comparable with G_3 which is the corresponding parameter for the more realistic elastic deformation model.

Coefficient of friction	μ_0	0.02 – 0.05
Yield stress	k	200 – 500 MPa
Strip entry thickness	h_i	> 0.005 mm
Work roll radius	R	80 – 150 mm
Work roll modulus	E	200,000 MPa
Strip speed	v	5 – 20 m/s

Table 1: Typical ranges of parameter values.

Nevertheless, the mattress approximation was pursued with the assumption that $G_3 >> 1$. Then, from (4.1) and (4.3a)

$$\frac{dh}{dx} = x + G_2\frac{dp}{dx}$$

and thus (4.2) becomes

$$(h - G_2)\frac{dp}{dx} + \hat{\mu}p = x \tag{4.5}$$

However, $h = O(1)$ and $\hat{\mu} = O(1)$ so that (4.5) with $G_2 >> 1$ is incapable of exhibiting the boundary layer behaviour observed in the IAS model. The cause of the inconsistency was thought to be the choice of length scale $\sqrt{Rh_i}$ and/or the assumption $K = O(1)$. Since it was clear that an analysis would not generalize to the more realistic elastic deformation (equation (4.3b)) the matter was not pursued further.

Since no large or small non-dimensional parameters were identified for the IAS model, the assumption of distinct inlet and outlet regions was questioned. Specifically, it was felt that equations (3.9) should be the equations for the inner and outer solutions of a singular perturbation problem and that a lack of a suitably small non-dimensional parameter meant that such simplifications could not be sustained. However a concensus on this point was not reached.

Another point questioned was the continuity of the pressure. A simple analysis suggested that there may be a logarithmic singularity at the point $x = b$ but again there was not a concensus on this point.

Finally, a number of proposals for numerical schemes were made but as these could not be demonstrated to be superior to the scheme employed by IAS, they are not reported here.

References

N.A. Fleck and K.L. Johnson, "Towards a new theory of cold rolling of thin foil", *Int. J. Mech. Sci.* **29** (1987), 507–524.

N.A. Fleck, K.L. Johnson, M.E. Mear and L.C. Zhang, "Cold rolling of foil", *Proc. Inst. Mech. Eng.* **206** (1992), 119–131.

K.L. Johnson, *Contact mechanics* (Cambridge University Press, Cambridge, 1984).