GAS FIELD SCHEDULING

Woodside Offshore Petroleum is the operator in the development of new gas fields in Australia's North West Shelf project. Sequencing the development of new gas fields in this project is a key determinant of its return on investment. This development sequence has constraints imposed by infrastructure and contractual obligations as well as natural features. The determination of an optimal or very good solution may involve a number of techniques from operations research.

The study group attempted several approaches to the problem, principal amongst them being mathematical programming and dynamic programming. A few other heuristic approaches were also considered. The mathematical programming approach was able to yield solutions to small instances of the problem. The group was able to identify several avenues for further research and work on the problem is ongoing.

1. Introduction

This problem was presented to the MISG by Woodside Offshore Petroleum (WOP), the operator in the consortium of companies which is spending \$25 billion over about 40 years developing gas fields off the West Australian coast near Dampier. The project, called the North West Shelf Project (NWSP), includes a large number of separate fields, each differing in size, development cost and hydrocarbon mix. Finding and developing a new field can cost upwards of \$2 billion and it can take 5 to 10 years before production can commence.

The problem facing WOP is to find the best feasible development sequence of the fields and the most efficient rate of their exploitation which ensures that existing contracts are satisfied and the overall return on the large investment is maximised.

The fields produce a variety of low rank hydrocarbons with other gases (mainly nitrogen and carbon dioxide) which are combined in different mixtures to produce natural gas (piped directly to the Western Australian domestic market as DOMGAS, and liquefied natural gas, for export to Japan as LNG), liquefied petroleum gas (LPG, a valuable by-product) and a light oil condensate (STC, stock tank condensate). In addition one of the fields at least (labeled D in this report) has conventional oil reserves. The hydrocarbon mix of individual fields determines the relative amount of each of these by-products that are produced. Long term contracts have already been agreed with buyers for DOMGAS and LNG sales. These contractual obligations have to be met.

Each potential field has a different character, in terms of gas composition. The composition of fields will change as they are produced (due to phase change behaviour), and the development costs for each field are different. The character of the fields that have been currently assessed is given in Table 1.

At present two fields have already commenced production. These are labeled A and C. Two more (labeled B and D here) are due to start in the next two years. In the future, there may be more than twenty fields, a number of which have already been identified.

Field Label	Gas Reserves (10^9sm^3)	Condensate Reserves (10^6sm^3)	CGR_{init} ($m^3/10^6$sm^3)	CGR_{fin} ($m^3/10^6$sm^3)	LPG Content (Tonnes/10^6sm^3)
A	182	21	130	101	51
B	56	25	522	371	175
C	15	2	220	47	69
D	7	1	157	129	459
F	27	6	258	186	106
H	52	10	258	127	106
I	16	2	220	30	69
J	54	13	292	189	99
K	13	7	674	403	145
L	5	3	674	526	222
M	21	5	258	218	106
N	12	3	365	135	106
O	10	3	393	207	106
P	18	2	168	54	71
Q	28	6	280	149	141
R	28	6	280	149	117
S	23	5	280	155	117
T	57	11	280	106	117
U	28	6	280	149	117

Table 1: Field characteristics.

Gas processing facilities have been installed on the Burrup Penninsula to treat gas from the field labeled A, which is situated 134 km offshore. The facilities remove impurities from the gas, such as carbon dioxide and water. Valuable by-products such as LPG and condensate are also removed. The processing plant is limited by the amount of sales gas products, condensate and LPG it can produce.

As can be seen from Table 1, the gas from the second field (labeled B) is much richer in LPG and condensate than gas from A. Hence, liquid production will increase substantially when field B is started. The problem becomes more critical as more fields are added to the project; each field with a different characteristic, gas composition and exploration cost. Moreover, the choice of development may not be limited to the fields shown in Table 1. Choices could come about through new discoveries in exploration permits or production licences. An added complication could arise if we include the possibility of purchasing third party gas, from parties not directly involved in the NWSP. Hence, the sequencing of developments and the extent of exploration in each of the developed fields will become very critical to the return on investment.

2. Modelling the problem

The gas and condensate reserves and the LPG content for the already identified fields are shown in Table 1. This information is used in determining the product yields for various mixes of field development. One factor affecting the solution is the steady reduction over time in the proportion of STC obtained from a field even though the field's gas yield stays constant. This is revealed in Table 1 which shows the disparity between the condensate to gas ratio (CGR) initially and finally (respectively, CGR_{init} and CGR_{fin}). The figures in Table 1 also reveal the STC–rich fields.

The CGR for any given stage in the life of a field can be found from the formula:

$$CGR = CGR_{init} - (CGR_{init} - CGR_{fin}) * CumulativeProduction/Reserves \quad (1)$$

Here, $CumulativeProduction$ represents the total amount of production up to that stage in the life of the field.

The capacity of a field is determined by the capacity of individual wells and the number of wells installed. Clearly, the capacity of a field can increase or decrease depending on the number of wells installed, which is a decision to be made in the case of fields that have not yet been developed. The capacity of the wells in each of the fields and their costs are supplied in Table 2. Until 70% of the reserves in a field are exhausted, the capacity of a field is found from the formula:

$$Capacity = WellCapacity * NoOfWells/1.5 \quad (2)$$

The field capacity then declines linearly until all the reserves are effectively depleted. (The field is terminated when a minimal uneconomic level of 3 million standard cubic metres per day is reached.)

The overall field development cost can be found from the data on startup and running costs (factors $Cost\_A$ and $Cost\_B$, respectively) in the last columns of Table 2.

The development cost is then given by the formula:

$$DevCost = WellCost * NoOfWells + Cost\_A + Cost\_B * Capacity \tag{3}$$

Field Label	$WellCapacity$ ($10^6 sm^3$/day)	$WellCost$ (Au$ m)	$Cost\_A$ (Au$ m)	$Cost\_B$ (Au$ m)
A	51	-	-	-
B	25	-	-	-
C	5	-	-	-
D	3	-	-	-
F	2.0	16	1	0.0
H	2.0	16	196	41.9
I	1.5	25	77	17.3
J	1.5	24	84	28.9
K	1.5	17	95	20.1
L	1.5	19	16	29.1
M	2.0	25	77	17.3
N	1.5	20	98	19.4
O	1.5	24	112	22.6
P	1.5	21	107	25.5
Q	1.5	24	139	26.9
R	1.5	26	155	28.7
S	1.5	20	128	9.8
T	1.5	24	82	15.0
U	1.5	26	155	28.7

Table 2: Well capacities and development costs

Note that, in the above table, $WellCost$, $Cost\_A$ and $Cost\_B$ for the fields labeled A, B, C and D have not been given. These fields have already been developed (or are under development). Also, the $WellCapacities$ for fields A, B, C, and D represent the overall capacities, while the figures against the remaining fields represent the capacities per well. The overall capacities for the remaining fields will be determined by deciding how many wells to install.

The first constraint to consider is the contract constraint. This fixes the net output of natural gas until the year 2009; the year until which long term gas contracts have already been signed. There are also upper limits on annual production of LPG and condensate related to export limits and processing facilities. The maximum condensate production rate is 16000 cubic metres per day and the maximum LPG production is 2.67 million tonnes per year. An additional long term constraint is that DOMGAS has priority over LNG and the production of the latter can only be wound down at a few stepped values: 31.8, 21.2, 10.6 or 0.0 million standard cubic metres per day.

Transporting the gas and oil to the marketplace involves a complex network of primary production points, interconnecting pipelines and drill platforms, points from which ships may be directly loaded at sea and on-shore processing facilities. One set of physical constraints is that gas flowing upstream through this network cannot exceed the capacity of the next installation in the chain. Clearly a precedence constraint dictates that a downstream facility cannot be exploited unless all links to the primary processing or loading points are operational. The links planned or possible in the currently identified fields are given in figure 1. The solid lines represent mandatory links and the dotted lines represent alternative links, which the model will choose in an optimal manner.

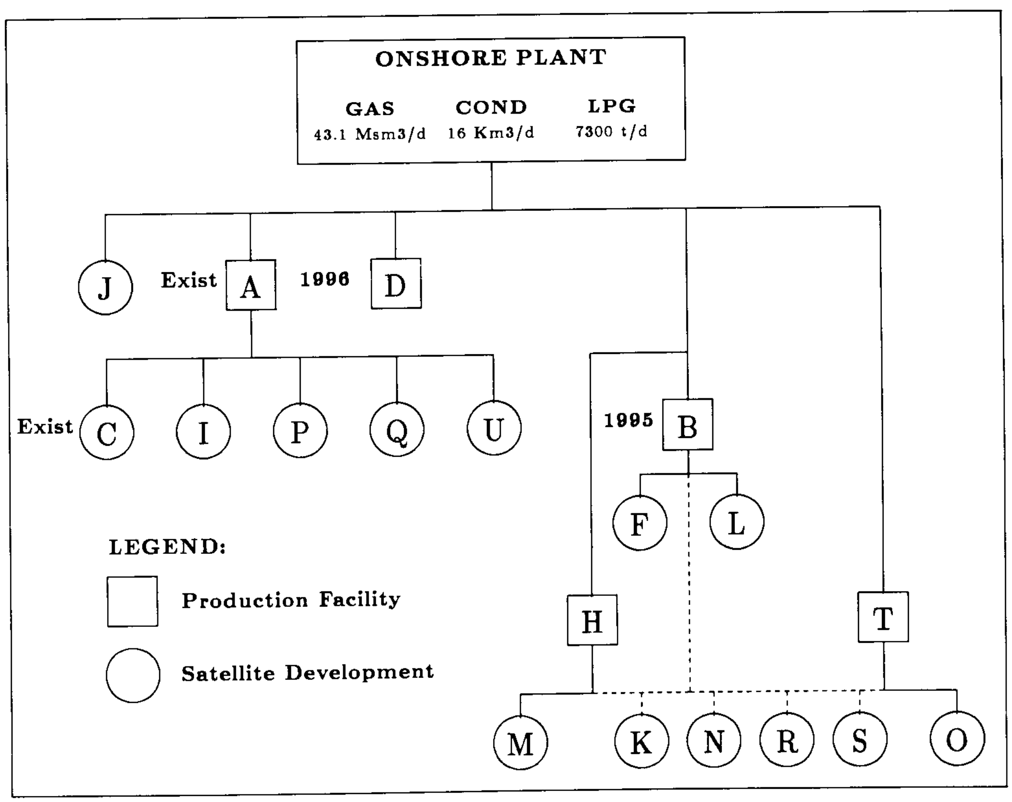

Figure 1: Proposed development network.

The present sequencing algorithm is a 'greedy' algorithm which ranks the exploitation order according to the *Unit Technical Cost*, or UTC, which is the present value of the development costs for the field, divided by the present value of the production. To maximise the return on the investment, it is necessary to use the *Net Present Value*, or NPV, instead. This accounts for the discounted cash flow of the collective investment over the lifetime of the project (about 40 years). This is a function of revenue and costs, which themselves depend on the individual field characteristics, tax rates, royalties and the capital cost environment.

The relevant prices for the products DOMGAS, LNG, Condensate and LPG are, respectively, $70,300 per million standard cubic metres, $110,900 per million standard cubic metres, $148 per cubic metre and $163 per tonne exported. The calculations assume a tax rate of 33%, a royalty of 12.5%, inflation of 5% and asset depreciation at 22.5% reducing balance.

The development expenditure is phased in over a six year period. 5% is spent in year 1, 10% in year 2, 20% in year 3, 25% in year 4, 25% in year 5 and 15% in year 6. Production starts in the fifth year and operational expenditures from then on are assumed to be 5% of the total capital expenditure.

The study group attempted two approaches to solve the problem, namely, mathematical programming and dynamic programming. In the former, the problem as stated above can be formulated as a large nonlinear mixed–integer program (NLMIP). Such problems are difficult to solve and solvable instances of such programs are small. Hence some simplifying assumptions are made to render the problem tractable. We describe the mathematical programming approach in Section 3. A dynamic programming approach formulated is described in Section 4. A few other approaches were also suggested, namely, neural networks, genetic algorithm, and interactive approaches. These are described in Section 5.

In the problem as given the formulation is deterministic but the data is subject to some uncertainty, so the sensitivity of solutions to data perturbations will be an issue. This avoids the formulation in stochastic terms which would most likely be intractable.

3. The Mathematical programming approach

Let I denote the set of gas fields, $I = \{i|i = 1, \ldots, M\}$, and J the set of years in the planning horizon, $J = \{j|j = 1, \ldots, N\}$. For each field i, define the ancestor set , $a(i)$, where $a(i) \equiv \{i'|$ field i' must be in operation prior to developing field i $\}$. Likewise, for each field i, define the descendent set $d(i)$, such that $d(i) \equiv \{i'|i \in a(i')\}$.

3.1 Problem formulation

Let x_{ij} denote the fraction of gas field $i \in I$ extracted in year $j \in J$.
δ_{ij} = 1 if field i is operational in year j
= 0 otherwise
w_{ij} = 1 if field i starts production in year j
= 0 otherwise.

The primary decision variables x_{ij} implicitly specify whether a field is operational or not. The secondary decision variables, δ_{ij} and w_{ij} depend on x_{ij} and are used in the evaluation of cost and revenue streams.

The original statement of the problem included the issue of determining the number of wells to operate in a field. This issue was not addressed by the mathematical programming formulation, because the resulting problem would become a nonlinear program. Nonlinear problems are generally difficult to solve.

We used an upper bound on the number of wells for the purpose of evaluating the development cost of a field, ($DevCost$) as given by (3).

Given the model data

g_i	Initial Gas reserve in field i
$R(x_{ij})$	Revenue from extracting x_{ij} percent of field i in year j
$D(w_{ij})$	Development cost if field i starts in year j – defined by (3)
$O(\delta_{ij})$	Operating cost accrued if field i is operational in year j
r	The annual discount rate
$GasDem_j$	Annual scheduled demand for gas in year j
LPG_{max}	Maximal permissible annual output of LPG
LPG_i	Percentage LPG content of gas extracted from field i
CGR_i	Condensate to Gas Ratio of field i
CON_{max}	Maximal permissible annual output of condensate
$\hat{x}_{ij}$	Maximal permissible extraction from field i in year j

The revenue from a field i in a particular year j is dependent only on x_{ij}, the percentage of gas extracted from it in that year. The development cost of a field i is dependent on the year j in which it becomes operational. This will be indicated by the value w_{ij}, which will take the value 1 when the field is started up. The operational cost for a field i in a year j is assumed to depend only on whether or not the field is operational in that year. This is determined by the value of δ_{ij}. For the sake of simplicity, we assume that all development expenditures are incurred in the year production commences. Moreover, we ignore, in this preliminary model, the impact of tax and depreciation.

Then our problem is to maximize the net present value of operations, NPV:

$$
\begin{aligned}
\max\ NPV &= \sum_{j=1}^{N}\sum_{i=1}^{M}((R(x_{ij}) - D(w_{ij}) - O(\delta_{ij}))/(1+r)^j \\
\sum_{j=1}^{N} x_{ij} &\leq 1 && && (4)\\
\sum_{i=1}^{M} x_{ij} g_i &\geq GasDem_j, && \forall j \in J && (5)\\
\sum_{i=1}^{M} 0.3 LPG_i\, g_i\, x_{ij} &\leq LPG_{max}, && \forall j \in J && (6)\\
\sum_{i=1}^{M} CGR_i\, g_i\, x_{ij} &\leq CON_{max}, && \forall j \in J && (7)\\
x_{ij} &\leq \delta_{ij}, && \forall i \in I, j \in J && (8)\\
x_{ij} &\geq -(1-\delta_{ij}) + \epsilon, && \forall i \in I, j \in J && (9)\\
w_{ij} &\geq \delta_{ij} - \delta_{ij-1}, && \forall i \in I, j \in J && (10)\\
\sum_{i=1}^{M} w_{ij} &\leq 1, && && (11)\\
x_{ij} &\geq 0, && \forall i \in I, j \in J && (12)\\
x_{ij} &\leq \hat{x}_{ij}, && \forall i \in I, j \in J && (13)\\
\delta_{ij} &\in \{0,1\}, && \forall i \in I, j \in J && (14)\\
w_{ij} &\geq 0, && \forall i \in I, j \in J && (15)\\
x_{ij} &\leq M * \textstyle\sum_{l=1}^{j-1} x_{a(i)l},\ \forall i, a(i), j \text{ such that } a(i) \neq \emptyset && && (16)\\
\sum_{l \in d(i)} x_{lj} g_l + x_{ij} &\leq \hat{x}_{ij} g_i, && \forall i, j && (17)
\end{aligned}
$$

Constraint (4) ensures that the total gas removed from any field doesn't exceed the total amount of gas present in the field. Constraint (5) ensures existing contracts are met. Note that we use inequality rather than equality constraints in (5). The optimal solution will result in more gas being produced *if* it is economically more attractive. Otherwise, the equality relationship wil hold. Constraint (6) imposes an upper bound on the amount of LPG that may be extracted in a given year. Note that this constraint implicitly includes WOP's system constraint that not more that 30% of the gas extracted from fields in a particular year can be converted into LPG. Constraint (7) bounds total annual condensate production.

Constraints (8) and (9) define whether a given field is operational in any particular year. The value ϵ is some small value, which may be used as a lower operational bound for the variables x_{ij}. Clearly, if x_{ij} is positive in a particular year, the value of δ_{ij} will be 1, if δ_{ij} can only take the values 0 or 1. This value can then be used in the objective function to determine the relevant operational cost of field i.

Constraint (10) checks whether a field is started in any particular year. If so, the corresponding value of w_{ij} becomes 1, if w_{ij} can only take the value 0 or 1. This can then be used to identify the relevant development cost of field i in the objective function. Constraint (11) forbids more than one startup per field.

Constraints (12) to (15) define the upper and lower bounds of the decision variables. Note that, the variables w_{ij} need not be explicitly defined as 0–1. The constraints (10) and (11) ensure that w_{ij} will always take the value 0 or 1.

Constraint (16) represents the network link constraint: a field may only be started if its predecessor (assuming it has one) is already in existence and is operational. The constant term $M \sim 10$ is present so as to prevent an artificial upper bound from being imposed.

Constraint (17) represents an upper bound on capacity handling: The sum of a given field i and its predecessors' total annual gas production must be less than the processing facility of field i.

A number of features are apparent in this formulation of the problem. Here the issue of setting up individual wells is not addressed. Information regarding the scheduling of well set-ups may be inferred from the optimal x_{ij} values.

The objective function represents a somewhat simplified and linearized version of the net present value of the project. It was, however, considered an adequate representation of the project.

Two important physical features have not been modelled. Firstly, it is assumed that the condensate to gas ratio remains constant for each field over the duration of extraction. In other words, the CGR, as defined by (1) was not modelled. Equation (1) involves $CumulativeProduction$, which is a function of x_{ij}. The inclusion of this formula in (7) would make the problem nonlinear. One possible approximation could be the inclusion of a piece-wise linear form of (1). However, we chose a constant CGR_i.

Secondly, as mentioned previously, the model does not consider the scheduling of wells and the well capacity. These simplifications ensured that the resulting model was linear and solvable.

3.2 Implementation and discussion of results

The formulation, as presented above (*i.e.*, without the nonlinear constraints) is a mixed integer linear program (MILP). The model was coded in the GAMS modelling language (see Brooke, Kendrick and Meeraus, 1988) and was solved using ZOOM, the integer programming solver.

The full problem, solved for $M = 19$ fields over an $N = 40$ year planning horizon, results in 760 boolean variables, 1341 continuous variables and 2860 constraints. Since it was not possible to solve such a large MILP using ZOOM, we relaxed the 0–1 integer restrictions on δ_{ij} and solved it as a relaxed mixed integer linear program (RMIP). Thus, δ_{ij} could take on a continuum of values in $[0, 1]$. This relaxation has implications for the nature of the solutions obtained, and will be discussed later.

Moreover, if we considered the costs as they are given, we found that the sequence contained what we describe as *holes.* Holes in a sequence represent a start–stop–start feature of production in a field. Clearly, this is undesirable. This is due to the relaxation of the 0–1 constraint on δ_{ij}. As a result, the w_{ij} variables also take on continuous values, hence permitting a number of start-ups to occur for the one field.

In order to avoid this, we impose an artificial penalty in the objective function in the term associated with the initial development cost. The resulting solution, presented in figure 2(a) and figure 2(b), does contain some holes. Also there is a tendency for a few fields to start up simultaneously (in the same year). For example, fields M, N and R commence production in the year 2012. This is due to the relaxation of the binary restriction and would disappear in the MILP solution.

As a further extension, it was desired to see the effect of modelling declining gas–condensate ratio upon the solution. That is, CGR_i in constraint (7) was evaluated using (1), thereby turning the problem into a mixed integer nonlinear program (MINLP). The integer restrictions were relaxed and the resulting MINLP was solved using GAMS–MINOS. The solutions produced were not too dissimilar to that depicted in figure 2(a) and figure 2(b).

4. The dynamic programming approach

Dynamic programming (DP) offers an exact method algorithm to determine the global optimum to the deterministic problem of sequencing the start-up times for each of the 19 fields, while simultaneously determining the production capacities for each field in each year of the planning horizon. The problem lends itself to a natural division into stages (years) where each stage can be determined uniquely from the previous stage only. Therefore, a DP approach is suited to this problem.

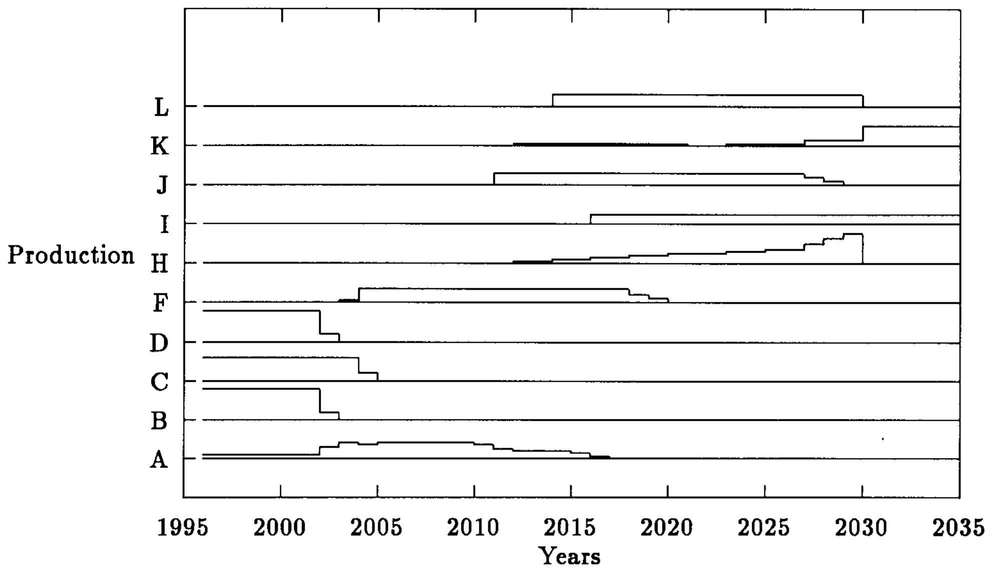

Figure 2(a): Mathematical programming solution (Fields A through L).

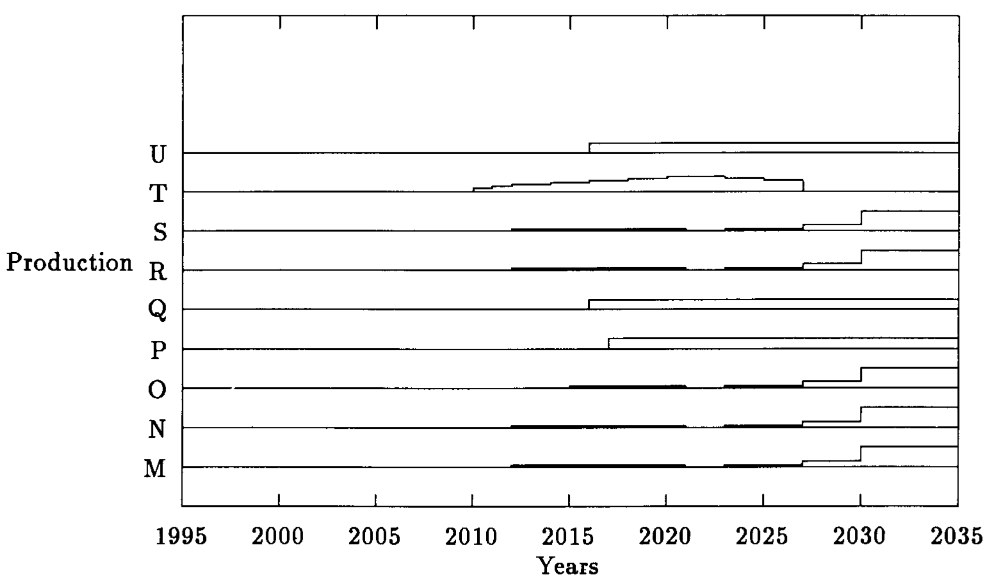

Figure 2(b): Mathematical programming solution (Fields M through U).

The main aims for each year (stage) of operation are to determine the fraction of each site's gas reserve to produce in that year, and the number of wells that must be drilled (if any) to meet the production capacity for that field in each year so as to maximize the NPV.

The number of wells required at each field in each year is determined uniquely by the production capacity of each field. It is important to keep track of the number of wells already existing at each field so that we do not double or triple cost the development of that well. For example, the production at field i (say) may require the following number of wells for 3 years of the planning horizon: (4, 3, 5). It is important to note here that in the third year we only need to cost the drilling of 1 well and not 2. The decision relating to wells is the only complicating factor in the DP formulation of this problem.

The main advantage of the DP method over all other methods studied is that it has no difficulty handling any of the nonlinearities in the profit functions, or any of the nonlinearities in the constraints on the operation of the gas fields. For example, it can easily handle the nonlinearities resulting from the accurate representation of CGR_i in constraint (7).

For use as the state variable for the DP we will introduce a new variable, Y_{ij}, the total fraction of g_i produced up to and including the j^{th} time period.

$$Y_{ij} = \sum_{k=0}^{j} x_{ik} \tag{18}$$

4.1 DP formulation

We shall suppress the time variable j and the state variable will be a vector,

$$\mathbf{Y} = (Y_1, \cdots, Y_i, \cdots, Y_{19}) \tag{19}$$

The decision at each stage (year) of operation is the amount of each field i to produce. Our decision variable is a vector $\mathbf{X}$, again with time j suppressed,

$$\mathbf{X} = (x_1, \cdots, x_i, \cdots, x_{19}) \tag{20}$$

If we let $F_J[\mathbf{Y}]$ represent the total (best) NPV from period 0 to J with a total fraction $\mathbf{Y}$ of gas reserve used then this problem can easily be modelled as a DP. The functional equation being,

$$F_J[\mathbf{Y}] = \max_{\mathbf{X}} \left\{ \left(\frac{1}{1+r}\right)^{J-1} CF + F_{J-1}[\mathbf{Y} - \mathbf{X}] \right\} \tag{21}$$

where CF represents the cash flow.

4.2 Algorithm

The following is a step procedure for carring out the DP.

STEP 1: Input required data, eg. gas reserves at each field, upper bounds on condensate and LPG production etc.

STEP 2: Initialize all vectors and start at period $J = 1$

STEP 3: Carry out DP

- i Loop through all possible state vectors (ie. all feasible combinations of the vector $\mathbf{Y}$).
- ii Loop through all possible decision vectors.
- iii Check feasibility of the $\mathbf{Y}$, $\mathbf{X}$ combination.
  - Check $\mathbf{Y} - \mathbf{X}$ does not result in any element being less than 0.
  - Check to see if gas reserves are less than 3.
  - Check constraints on maximum condensate and LPG production.
  - Check the precedence relationships
  - Check that the capacities of each platform ie. field capacities A,B,C,D have not been exceeded.
- iv Calculate the benefit of this combination if feasible, if the combination is not feasible then assign a large negative benefit.
- v Goto ii) if there are more decision vectors to consider.
- vi Select maximum benefit for this $\mathbf{Y}$ combination, goto i) if more state vectors to consider.

STEP 4: Move to year $J + 1$, goto Step 3 until the end of the planning horizon has been reached.

4.3 Dimensionality

This formulation is very simple but we have a problem with the size of the state space. If we consider potential increments for each element in $\mathbf{Y}$ of 0.001 then for the 19 field problem the number of potential states is

$$\approx 1.019 * 10^{57}$$

This problem is known in DP as *the curse of dimensionality.*

4.4 Potential methods for overcoming dimensionality

The main method of dealing with this problem is to reduce the size of the state space. The following are four potential methods for achieving this.

1. Instead of considering all possible combinations of the vector $\mathbf{Y}$ at each time period, we can make use of the conditions of the problem to consider only the feasible state vectors $\mathbf{Y}$ in each time period.

2. Reduce the number of fields by grouping together fields with similar characteristics into one field, thereby reducing the problem to, say, a four field problem. This can be optimized using DP. Then, with the known production requirements for each of the sites optimize within each site to determine the production capacities of each field.

3. The most promising method is to use the corridor method. This method involves starting with an initial feasible solution (possibly the one WOP already have) and to consider a region of perturbation around this (defined as a corridor). The state space can be defined as this region. Carry out DP in this space and look for improvements. Repeat the process until no further improvements can be found, and a local maximum has been located.

4. Another promising method that should be investigated is the application of *nonserial DP* to this problem (see Nemhauser, 1966). Nonserial methods provide for situations involving a number of serial subprocesses that are linked in a variety of combinations. For the WOP problem, we can see from the facility tree that there are essentially two main regions: Region 1 containing fields A,C,D,I,J,P,Q,U and Region 2 containing all other fields. Nonserial DP will determine the optimal method for producing $0, 1, 2, \cdots$ units of gas in each region in each year. The regions are then combined to meet, optimally, the requirements of the onshore plant. This problem may require the use of more than 2 regions, but the method explained above remains the same.

Methods 2 and 3 do not guarantee a global maximum but they may generate a solution which is better than the one WOP have at present.

5. Other approaches

The first of these is an *interactive LP approach.* In Section 3, the mathematical programming formulation to the problem led to a MILP with many 0–1 variables. The complexity introduced by these variables can be handled by interactive use of a LP package. Consider fields A, B, C, ..., with A already operating. A setup cost is incurred in the first year of operation of fields B, C, ... Denote by $x_{Aj}, x_{Bj}, \ldots$, the amount of gas extracted from fields A, B, ..., in year j. Let $c_{Aj}, c_{Bj}, \ldots$, be the marginal costs. Suppose that an LP calculation (without the setup costs) starts B, say, in year k and let the total cost be F_1. This may be compared with another LP calculation with c_{Bk} replaced by $c_{Bk} + S_B/x_{Bk}$, if S_B is the setup cost. Let the cost of this solution be F_2. This may defer startup of B to a later year, say l. Startup of B in year k would be preferred if $F_1 < F_2 + S_B$. If not, another calculation has to be made with modified c_{Bl}, with the possibilities of startup in earlier years excluded by introducing penalty costs. Note that benefits would enter here as negative costs. On the assumption that extraction from a field becomes somewhat less profitable in later years, extraction would start in the first year indicated as profitable by the interactive calculation, and then continue for an uninterrupted period of years, till supply runs out in the field. However, a global optimum is not guaranteed by this procedure.

This approach is related to *separable programming,* where a nonlinear function of a single variable is approximated by a piece-wise linear function. To treat setup costs in this way, the variable $x \geq 0$ is replaced by a *special ordered set* of three variables to describe a cost function graphed by two line segments, with two specified variables not allowed to occur together in a basis. This requires some modification to an LP code — see Beale (1970). But the computation is much the same as that described for the interactive LP approach described above. Moreover, as the model is non-convex, the approach is likely to stick to a local optimum, unless some assumptions are made about costs as a function of time.

There are probably additional constraints for the real problem that have not been featured in the mathematical programming and the DP approaches. Namely, limits on resources would ensure that only a few fields were started up or run at the same time. With this in mind, only small regions of the solution space need to be explored at any one time. This suggests an interactive use of GAMS (or any other MILP package that is used). So we could perform a GAMS run with fields A and B open, another with just A and C open, and so on. After examining the results, we could pick out several candidate fields, say X,Y,Z, for exploitation. Earlier startup years can be excluded by penalizing the marginal costs, so only a few 0–1 variables (corresponding to startup years) need enter the next round of calculation, say, considering A,X,Y or A,X,Z. The above interactive approaches would stand a better chance of solving the entire problem along with all the complicating system constraints. Firstly, it allows input of practical wisdom. Secondly, it escapes potential computational problems.

Two other heuristic solution approaches were considered. The first of these was neural networks. However, the solution of the problem via this approach requires good sample solutions to enable the network to learn and adapt and optimize when faced with new situations. Although it was thought that the neural network would work reasonably well, the absence of good initial solutions precluded the use of this approach. The second approach involved genetic algorithms (GA). However, it was soon found that it would be very difficult to obtain an efficient representation of the problem, which is crucial to the success of any GA approach. Moreover, it was felt that the calculation of the objective value after each successive iteration of the GA would be computationally expensive. Hence, these novel approaches were not explored further.

6. Conclusions

The study group identified several approaches for solving the problem of sequencing field start up times for WOP. Of the approaches that were suggested, the mathematical programming approach achieved maximum progress, although only a simplified version of the problem was solved. The full model would include the declining gas–condensate ratio feature, the tailing off behaviour and the phase change behaviour in the gas reserves, and an accurate representation of cost streams. Such a model would turn the problem into a MINLP which is difficult to solve. The problem would then require specially tailored solution procedures and special-purpose codes would need to be written. This would take the solution procedure out of the domain of GAMS solvers. However, the GAMS approach has yielded prototype solutions that look promising.

Further work needs to be done in order to determine appropriate solution approaches. Moreover, we need to address the issue relating to well capacities and well start-up times. A good understanding of decomposition solution methods can be useful in solving the large MINLP without having to resort to methods that relax the integer restrictions. The problem shows a great deal of parallelism, with blocks of constraints of similar structure appearing for each field. Advantage could be taken of this correlation via decomposition methods. Also, the expected highly correlated nature of the δ_{ij} variables suggests that an intelligent branch and bound procedure may be developed that exploits the structure of the problem.

The DP approach was formulated and refined to a great extent. However, not much progress was made due to the lack of computing resources and to the *curse of dimensionality*. Several interesting modifications and approaches have been highlighted and further work in this area is definitely possible.

Constrained Logic Programming (CLP) is one approach that was not considered by the study group. This method performs a systematic search of the problem's feasible space to obtain solutions. The problem and the search space can often be specified in a simple and elegant manner. The ease of applicability of this approach to the Woodside problem is an open question and needs to be investigated.

There are several promising approaches, highlighted in Section 5, which can be followed up to produce good interactive LP-based heuristic solutions.

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