SCHEDULING IN THE MANUFACTURE OF EVAPORATIVE AIR CONDITIONERS

The MISG examined the problem of scheduling production of air-conditioners at Seeley International. Seeley's objective was to meet their demand in a more cost-effective way. Two models are proposed to achieve this objective. A long term master production schedule with a yearly planning horizon was formulated to give Seeley a broad-based schedule for planning production to meet forecast demands and production constraints. Output from this model is designed to provide the appropriate number of units of each product type or subassembly item to be produced in the following week. This output is then designed to be the input to a more detailed short term model for scheduling production at the machine level. The short term model is formulated to handle a mixture of both finished products and subassemblies. The objective of the short term model is to minimise total production time to free up the use of resources in order to allow for external orders. Directions for further work are discussed.

1. Introduction

Seeley International (SI) is a major manufacturer of air conditioners in Australia, dominating the Australian evaporative air conditioner market under the brand names Convair and Breezair. In addition to supplying the Australian market, they have a growing export market, currently comprising over 40% of their business. The company has an annual growth rate of 15% and a predicted growth rate of 20% for the season 1996/1997.

Manufacturing is based at St Marys in Adelaide, where units are fully assembled and from where they are distributed to warehouses in Perth, Melbourne, Sydney, and Brisbane, in addition to their National warehouse in Adelaide. Over 70 different model options are available, giving variations in the colour of the unit (4), the motor type (13), cabinet size (5), and the type of discharge (3). Although not all combinations of colour, motor, cabinet, and discharge are available, the provision of around 70 variations is a reflection of a highly competitive market in which satisfaction of individual customer requirements is of prime importance. It is SI's desire to respond promptly to this market that initiated the MISG study.

In their current situation SI takes orders from around 130 air conditioner dealers nationwide. Until recently these dealers have been used to ordering 24 hours prior to installation, since traditionally stock has been available at their respective state warehouse. In hot summers, such as in 1994/1995, rapid increases in demand make it difficult to schedule production to meet demand, particularly when considering the range of products, and the need to maintain warehouse supplies interstate. Whilst supply problems could be fixed by stockpiling of finished products in the off season, or by sinking more capital into a new plant, neither option is appealing since neither is efficient nor cost effective in the long term. For this reason SI turned their attention to scheduling.

SI asked the MISG to consider the scheduling of their manufacturing operation in order to satisfy a number of objectives, viz.

- To guarantee customer satisfaction 100% of the time by delivering on time;
- To respond to urgent requirements;
- To offer all possible model options;
- To optimise the production capacity of the existing manufacturing facility with the minimum capital expenditure;
- To provide a sound basis for forecasting;
- To minimise inventory of WIP and finished stock;
- To eliminate waste associated with urgent purchases and deliveries.

Although not listed as an objective, it was implicit in SI's requirements that the MISG should work within the scheduling framework currently used. In addition, Sean Fitzpatrick from SI came to the group with a number of ideas. One of these was to schedule by subassembly rather than by finished product. We will develop this concept further before discussing other issues.

It was proposed that by scheduling subassemblies rather than finished products, SI would be better able to respond to variation in product demand. Since many different subassemblies (motors, cabinets, blowers, etc.) are common to a number of different finished products, by having a stock of such subassemblies, rather than a corresponding limited range of finished products, SI would be better able to respond to demand variations. In reality a combination of finished products and subassemblies would be scheduled on any given day. The subassembly idea was embraced by the MISG team, and supported by a simulation using just four finished products and five subassemblies. A more detailed analysis of subassemblies and their relation to finished products was then considered and is described in Section 3.

A number of issues were discussed by the group in the process of coming to an understanding of the problem. It was decided at an early stage in these discussions that forecasting aspects of the problem would not be considered, but the group would concentrate on scheduling issues. Among several ideas and issues discussed was the idea of separating market demand into 'baseline' and 'summer variation' components. It was suggested that the baseline component could be satisfied by making finished product to stock, and that the variable component could be satisfied by subassemblies. It was conjectured that subassembly demand was more stable and predictable than finished product demand.

In keeping with the group's implicit requirement to work within SI's existing scheduling framework, it was decided to adopt a two phase approach to scheduling. The first phase involved developing a long term master production schedule with a yearly planning horizon. This schedule was driven by forecast demand, and within the most imminent weeks provides a basis for a more detailed schedule for production at the machine level, which may involve a combination of final products and subassemblies — the so-called Short Term schedule.

Dynamic programming was suggested as a possible modelling tool for the long term model, however it was decided that this approach may not be able to handle the model complexities and dimensionality, and hence the group opted for the more traditional linear programming approach. The literature contains a large number of papers describing scheduling in a variety of manufacturing environments. A recent treatise giving a thorough overview of the state of the art for scheduling problems similar to the SI problem can be found in Graves *et al.* (1993).

Not surprisingly however, no specific account is available which precisely addresses the issues discussed here. In Section 2 we give a detailed account of the long term model as developed by the group. In Section 3 we discuss an outline of the short term model, and finally we suggest future directions which SI may take to create an integrated scheduling system.

2. The long term planning model

2.1 Introduction

As discussed in Section 1, a long term production planning model is required to provide SI with a master schedule on which to base the management of production over a yearly time horizon. This model must also provide a link to scheduling requirements in the most imminent week. In other words, the schedule prescribed by the long term model for the following week's production will be used as input to a more detailed short term model at the machine level for each individual product. To accommodate this requirement, schedule periods for the long term model are chosen to be *days* for the most imminent last five days, *weeks* for the next seven periods, and *months* for the last eleven periods. This allows for a schedule horizon of thirteen four-week 'months', or one year in total.

The modelling approach discussed by the working group was a demand driven time staged linear program. The objective was to minimise the total cost of production which includes unit production costs, inventory holding costs, the cost associated with failure to meet demand, and finally, a penalty for variation in production levels between adjacent periods. The constraints include standard inventory conservation equations and others which will be discussed in detail shortly.

The use of a stochastic model to accommodate SI's use of three budget forecasts for 'normal', 'hot', or 'cool' summers was discussed by the group, and may be considered for future implementation, but it will not be presented here.

2.2 Notation

Indices:

$i = 1 \dots N$ An index for each item type to be produced.

$t = 1 \dots T$ The period during which production occurs.

Integer Variables:

x_{it} The level of production of item type i during period t.

y_{it} The inventory level of item type i at the end of period t.

u_{it} The unmet demand for item type i at the end of period t.

Continuous Variables:

s^1_{it} The production rate decrease of item type i from period $t-1$ to period t.

s^2_{it} The production rate increase of item type i from period $t-1$ to period t.

Parameters:

l_t The length of period t.

c_i The production cost per item for item type i.

h_{it} The inventory holding cost per item per period for item type i in period t.

Parameters:

o_i	The out-of-stock cost (penalty for unmet demand) per item for item type i.
D_{it}	The demand for item type i in period t.
$Y_i^{\min}$, $Y_i^{\max}$	The minimum and maximum allowable inventory levels for item type i.
Y_i^0	The inventory level of item type i at the start of the planning period.
X_i^0	The production rate of item type i just prior to the start of the planning period.
p_{it}	The production resource used per item of item type i produced in period t.
$P_{\min}(t)$, $P_{\max}(t)$	The minimum and maximum allowable production resource utilisation in period t.
v_t	The penalty per unit variation in production rate from period $t-1$ to period t.
g	The projected growth in demand for one year hence, as a fraction of the total demand.

2.3 The model

The objective is to

$$\text{Minimise:} \qquad \sum_{t=1}^{T}\sum_{i=1}^{N}\{c_i x_{it} + h_{it}y_{it} + o_i u_{it} + v_t(s_{it}^1 + s_{it}^2)\}.$$

Subject to the following constraints:

1. Inventory balance

$$y_{it} = y_{i,t-1} + x_{it} - D_{it} + u_{it} \qquad i = 1\ldots N\ ,\ t = 1\ldots T.$$

2. Initial inventory levels. These are the stock levels for each product at the beginning of the planning period. They are updated each time the model is re-run.

$$y_{i0} = Y_i^0 \qquad i = 1\ldots N.$$

3. 'Circular' inventory constraints. If the model does not contain conditions which set final inventory levels, the model will tend to force end of year stock levels as low as possible, at the same time tending to produce a lot of items at the start of the year. We require a constraint which avoids this, and at the same time allows for growth in demand, with a commensurate growth in the initial inventory required for the following year. One possible model which assumes a linear relationship between demand and inventory is

$$y_{iT} \geq \lceil (1+g)Y_i^0 \rceil \qquad i = 1 \ldots N.$$

4. Production capacity. Production in each period will be constrained both above and below by workforce and capacity limitations. Hence we have

$$P_{\min}(t) \leq \sum_{i=1}^{N} p_{it} x_{it} \leq P_{\max}(t) \qquad t = 1 \ldots T.$$

5. Production smoothing constraints. Large variations in production in adjacent periods have serious repercussions on workforce requirements. Two alternative methods for reducing production variations between adjacent periods were considered. The first approach involved applying constraints which specified limits such as, no more than a 10% variation between adjacent production periods. In fact such variations can be chosen selectively in specific period sets, such as say 10% in the first 5 days and 20% thereafter. Whilst this approach worked well, there was no effective way of trading off these variations with other costs. Consequently, an alternative method was applied which effectively allows such variations to find their own level by applying a suitably chosen penalty in the objective. Thus we apply the following constraints:-

$$\frac{1}{l_t} x_{it} - \frac{1}{l_{t-1}} x_{i,t-1} + s_{it}^1 - s_{it}^2 = 0 \qquad i = 1 \ldots N \; , \; t = 2 \ldots T$$

with the boundary constraint

$$\frac{1}{l_1} x_{i1} - X_i^0 + s_{i1}^1 - s_{i1}^2 = 0 \qquad i = 1 \ldots N.$$

The variable s_{it}^1 can be regarded as the decrease in production rate from period $t-1$ to period t, whilst s_{it}^2 is the increase in production rate from period $t-1$ to period t. Since periods are of varying length (days, weeks, months), the production levels x_{it} are divided by l_t to give a normalized production rate. Boundary conditions assume a prior production rate of X_i^0. The nett change in production rate from period $t-1$ to t is given by $s_{it}^1 + s_{it}^2$, which is penalized in the objective function with penalty v_t.

6. Inventory holding levels. SI requires both lower and upper limits on inventory levels. Minimum levels are required to ensure stock is on hand, whilst maximum levels are determined by physical space limitations. Thus we have:-
$$Y_i^{\min} \le y_{it} \le Y_i^{\max} \qquad i = 1 \ldots N \; , \; t = 1 \ldots T.$$

7. Nonnegative variables: $x_{it}\,,\, y_{it}\,,\, u_{it}\,,\, s_{it}^1\,,\, s_{it}^2 \ge 0$.

Note that this model is an integer programming problem. However if constraint 4 is omitted, then the model decomposes by product type, and if constraint 5 is also omitted, the model for each product type is equivalent to a network flow problem. Hence if constraints 4 and 5 are omitted, the solution to the linear program formed by relaxing the integrality restrictions will in fact be integer. However, if either constraint 4 or constraint 5 is included in the model, the linear programming solution may not be integral. Although this is of concern in theory, in practice we would expect that the inventory and production levels would be large, and the problem not tightly constrained, thus rounding of the linear programming solution would yield good solutions to the integer programming problem.

2.4 Results

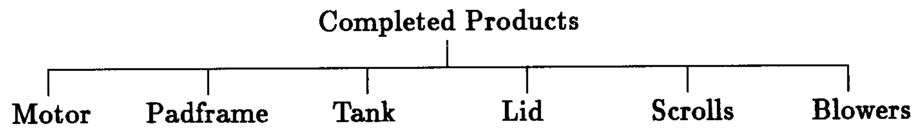
The long term model was validated against a limited data set provided by SI at the MISG. Products were grouped into eleven categories according to motor type. Demands over a twelve month planning horizon were aggregated for each category, based on historical data and current forecasts. Approximate production costs and inventory holding costs per unit per month were supplied by Sean Fitzpatrick, as were estimates for minimum and maximum inventory levels, initial inventory levels, and production capacity. Penalty costs for stock-out were based on average unit production costs. The model was initially run over a 12 month planning horizon. Subsequently the planning horizon was segmented into 23 periods, the first 5 representing the most imminent week, the next 7 the following 7 weeks, and the last 11 the remaining four-weekly periods, giving in total a planning horizon of 13 four-week months. Initial results indicate that the long term model behaves as expected, providing realistic figures for production and inventory over each period. The addition of production smoothing constraints followed the observation that whilst production figures in isolation appear realistic, there are fluctuations between adjacent periods which would not be acceptable in practice. Application of these constraints to the model yield appropriately smooth production with little or no change to the final objective.

3. Modelling of the short term scheduling problem

3.1 Introduction

3.1.1 Description

Seeley products are assembled on a final assembly line from six different subassemblies, shown in the diagram below. A simplifying and realistic assumption is that the manufacture of each subassembly is independent of the others; production of a given subassembly uses dedicated machinery and personnel.



All subassemblies come in different types. For example, padframes come in three sizes and four colours in the EA model, and in two sizes in the EM model, thus there are fourteen different types of padframe.

Completed products come in models, determined by the mix of different subassemblies. For example, model EM105DV uses an EM105 motor, a medium padframe, etc.

3.1.2 Outline of modelling approach

The Short Term Scheduling Model will be built in two phases. In the first phase, we model the problem as if *any rate of subassembly production* and *any rate of final assembly* could be achieved. This leads to variable definitions involving final assembly levels, subassembly production levels, and inventory levels for both completed products and subassemblies.

However we must constrain production and assembly levels so as not to exceed machine or assembly-line capacities. In the second phase we suggest how this might be done *without* explicit modelling of machine schedules. We feel that if it is possible to capture the essential features of the production process *without* constructing a machine schedule, then this must be done; otherwise there is a risk that the model would become extremely complex and difficult to solve.

The second phase models the time required on each machine or assembly line to meet the specified production level. This permits us to constrain production levels so that they can be achieved in the time available on the machine. Furthermore it allows us to develop an objective function which maximises machine idle time. This is considered desirable, since idle time can be used for external production, thus increasing revenue.

The second phase presents in a general way how machine time can be determined simply from production levels. Three examples of situations in which this may be done are given. It is expected that all production and assembly could be modelled, at least approximately, by one of these. Further research is needed to either confirm that this is the case, or to determine what other factors need be considered.

In combination, the first and second phases provide a skeleton model for the Short Term Scheduling Problem. However many specifics must be addressed — especially in relation to machine capacity — before a truly realistic model can evolve.

3.2 Phase 1: Production, assembly and inventory levels

3.2.1 Inputs

1. Time frame, e.g. day 1 to day 14.
2. Number of completed products that must be produced, broken down by size, colour, etc. (all the factors that determine the model) and the latest time period by which they must be ready.

3.2.2 Outputs

1. Number of subassemblies of each type produced in each time period.
2. Number of completed products of each model assembled in each time period.

3.2.3 Data

$$
\begin{aligned}
M &= \text{number of models} \\
S &= \text{number of subassemblies} \\
n_s &= \text{number of types of subassembly } s, \; s = 1, \ldots, S \\
T &= \text{number of time periods in the schedule} \\
D_{mt} &= \text{number of completed products of model } m \text{ due to be ready by} \\
&\quad\ \text{time period } t, m = 1, \ldots, M, \; t = 1, \ldots, T \\
W_{sk} &= \text{number of subassemblies } s \text{ of type } k \text{ available at the start of} \\
&\quad\ \text{time period } 1, s = 1, \ldots, S, \; k = 1, \ldots, n_s \\
A_{msk} &= \begin{cases} 1, & \text{model } m \text{ uses subassembly } s \text{ of type } k \\ 0, & \text{otherwise} \end{cases} \\
&\quad\ m = 1, \ldots, M, \; s = 1, \ldots, S, \; k = 1, \ldots, n_s
\end{aligned}
$$

3.2.4 Dimensions

Since the subassemblies are motors, padframes, tanks, lids, scrolls and blowers, we have $S = 6$.

From the illustration given in the problem description, we know that there are fourteen different types of padframes, so if padframes are the subassembly indexed by 2, then $n_2 = 14$.

We know the following quantities approximately:

$$\begin{aligned} \sum_{s=1}^{S} n_s &\approx 53, \\ T &\approx 14 \rightarrow 21 \text{ days, and} \\ M &\approx 100. \end{aligned}$$

3.2.5 Assumptions

1. Any subassembly can be stored and recalled at a later date for assembly without cost or extra time.
2. A subassembly produced in time period t can be used in final assembly in period $t + 1$ and not before.
3. All subassembly production is for the purpose of scheduled assembly, i.e. subassemblies are not produced or scheduled independently.
4. Demands D_{mt} must be met through current production, i.e. demand to be met by production completed prior to the start of the schedule has been subtracted.
5. Demands were determined through a long term scheduling model, and may include production for warehousing purposes in addition to immediate demand.

Note: Assumption 3 can be easily relaxed with minor modifications to the model.

3.2.6 Variables

x_{mt} = number of completed products of model m assembled in period t

y_{mt} = inventory of completed products of model m at the end of period t

z_{skt} = number of subassemblies s of type k produced in period t
w_{skt} = inventory of subassemblies s of type k at the end of period t

If the data in the problem is integer (as is expected) then it will be seen to be sufficient to constrain x_{mt} and z_{skt} to be integer for each $m = 1,\ldots,M$, $s = 1,\ldots,S$, $k = 1,\ldots,n_s$ and $t = 1,\ldots,T$. If these variables are integer, then the other variables will be integer without further constraints. From the dimensions given in Section 3.2.4, we conclude that the model has of the order of $MT + T\sum_{s=1}^{S} n_s \approx 3213$ integer variables, and the same number of continuous variables. This represents a large integer programming problem, and thus our concern is to develop production constraints which are as simple as possible.

3.2.7 Production balance equations

$$\begin{aligned} y_{mt} &= y_{m,t-1} + x_{mt} - D_{mt} \\ y_{m0} &= 0 \end{aligned}$$

$$\begin{aligned} w_{skt} &= w_{s,k,t-1} + z_{skt} - \sum_{m=1}^{M} A_{msk} x_{m,t+1} \\ w_{sk0} &= W_{sk} - \sum_{m=1}^{M} A_{msk} x_{m1} \end{aligned}$$

3.3 Phase 2: Machine scheduling and production capacity

Note that under the assumptions given below, models address the problem of a single product having different types, each with a specified production level which is to be met by production in a single time period. All models presented in this section can apply equally well to final assembly as to subassembly production.

3.3.1 Assumptions

1. Production of each subassembly $1,\ldots,S$, and final assembly, have independent capacity. Thus we drop the subscript s, and refer simply to z_{kt} to refer to the production level of product type k in period t. For simplicity, we may also drop the subscript t; it is clear from the context where t ought to be applied.

2. All products are manufactured on a single machine, and time available on the machine is denoted by U.

3. If several different product types are manufactured in a single period, then they are scheduled in the most time-efficient manner. In particular, this means that if $z_k > 0$ units of product type k are to be made, they will be manufactured in an uninterrupted block.

3.3.2 Model 1

If production on a machine can switch between product types without penalty, i.e. without requiring extra time, then we may use an extremely simple linear model. Note that it is believed that such a model may well approximate final assembly.

$$p_k = \text{production time per unit of product type } k$$

Machine capacity constraint:

$$\sum_{k=1}^{n} p_k z_k \leq U.$$

3.3.3 Model 2

The simplest nonlinear situation is that in which each product type has a fixed setup time on the machine.

$$r_k = \text{setup time for production of product type } k$$

Introduce the binary variables

$$\delta_k = \begin{cases} 1, & z_k > 0 \\ 0, & z_k = 0 \end{cases}$$

for $k = 1, \ldots, n$. This logical relationships may be modelled by the constraints

$$\delta_k \geq \frac{p_k}{U} z_k, \qquad \delta_k \in \{0, 1\}$$

for all k.

Machine capacity constraint:

$$\sum_{k=1}^{n} (p_k z_k + r_k \delta_k) \leq U.$$

3.3.4 Model 3

This model takes Model 2 and adds a level of complication. To illustrate, we note that for some products, the product types can be divided into two classes, such as EA and EM. If some product types from both classes are to be manufactured, an extra setup time, R, is required.

Let $C_1 \subset \{1,\dots,n\}$ and $C_2 = \{1,\dots,n\} \setminus C_1$ be the two classes. We introduce several binary variables:

$$\beta_i = \begin{cases} 1, & \sum_{k \in C_i} z_k > 0 \\ 0, & \sum_{k \in C_i} z_k = 0 \end{cases}$$

for $i = 1, 2$ and

$$\gamma = \begin{cases} 1, & \beta_1 = 1 \text{ and } \beta_2 = 1 \\ 0, & \text{otherwise.} \end{cases}$$

These logical relationships may be modelled by the constraints

$$\beta_i \geq \frac{\sum_{k \in C_i} p_k z_k}{U}, \qquad \beta_i \in \{0,1\}$$

for $i = 1, 2$ and

$$\gamma \geq \beta_1 + \beta_2 - 1, \qquad \gamma \in \{0,1\}.$$

Machine capacity constraint:

$$\sum_{k=1}^{n} (p_k z_k + r_k \delta_k + R\gamma) \leq U.$$

3.3.5 Objective

To maximise idle time on the machines, we may simply maximise the slack in the machine capacity constraints. If $1,\dots,C$ denote the C capacity constraints, with slack variables $\Delta_1,\dots,\Delta_C \geq 0$, our objective might be

$$\max \sum_{i=1}^{C} \Delta_i$$

4. Conclusions

The MISG addressed SI's objectives by formulating two mathematical programs.

The first will assist SI to best offset inventory holding costs versus production costs in their long term production schedule. The model proposed is general in the sense that either final products, or subassemblies, could be scheduled under the same model by using appropriate datasets. In the latter case, the final products required would be broken down into their subassemblies, and the total demand for each subassembly calculated. These would become the product demands input to the model. Since the model is well approximated by a linear program, it will solve extremely quickly for the size of problems that SI have. This will allow SI to develop production schedules for a range of possible demand scenarios, and thus enable SI to plan against demand uncertainty. There is the potential for the model to be re-formulated in a way which takes account of uncertainty in a more formal way. That is an avenue which further work may explore. As it stands, the long term model provides SI with a fast and flexible planning aid, which could be used in a variety of ways. In addition to providing the basis for a cost-effective production schedule, the long term model may be used to compare and contrast different production strategies, such as maintaining subassembly inventory in addition to fully assembled products.

The second mathematical program schedules production at a much more detailed level than the first. Product demands arising from the long term schedule drive production; they are devolved into demands for subassemblies, and final assembly requirements. The short term model attempts to schedule production of subassemblies in the most time-efficient manner, i.e. so as to maximise idle time of machines. The purpose of this is to free up machine time, which can then be used in revenue generating activities. The difficulty with scheduling subassemblies lies in determining whether or not a given production level is feasible. This is a very challenging aspect of the modelling process. The MISG group developed three general frameworks, for three types of subassembly production having different characteristics. The MISG group, however, did not have the opportunity to test these short term models with real data, and further work is needed to verify that these models are appropriate for all types of production that SI undertakes. Furthermore, the complexity of the short term scheduling problem leaves open the question of how best to solve it, which would also be a direction for future research.

The MISG work has provided a mathematical basis for an efficient production scheduling system for SI which directly addresses their objectives of optimising the production capacity of the existing manufacturing facility with the minimum capital expenditure, and minimising inventory of WIP and finished stock. It has the flexibility to assist them in improving their approach to achieving other objectives. Directions for further work would include a more detailed short term model, a more formal approach to demand uncertainty, implementation of the short term model using mathematical programming software, and the development of solution procedures.

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