A Medical Waste Sterilizer

Problem Presenter

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Problem Statement

Sterilization of medical waste is very important for the environment, as the exposition may result in various diseases because of viral or bacterial content of the waste. There are devices developed for this purpose aiming to sterilize the waste in a form called batch process, meaning that a certain amount of waste is placed into the system and subjected to a sterilization procedure for a while and removed from the system afterwards. The procedure is repeated by the next set of waste till the whole set is sterilized.

Our aim, however, is to design a device, a rotating cylinderical container having tubular lights attached to the walls inside, through which the waste is exposed to ultraviolet light as it gets rotated and moved towards the exit. The process will continue till the whole set is fed into the system. Such a device would be more effective as compared to batch processing types. The study group is asked to develop a mathematical model to analyse the effect of the number and location of tubes which will lead to maximal exposure during certain amount of time, which also needs an estimate, the sample will reside in the device before it gets discharged.

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Content

1. Problem Statement

The company would like to design a device as indicated in Figure 1. Medical waste is to be continuously fed into the device from the top and follow a helical path as it moves down.

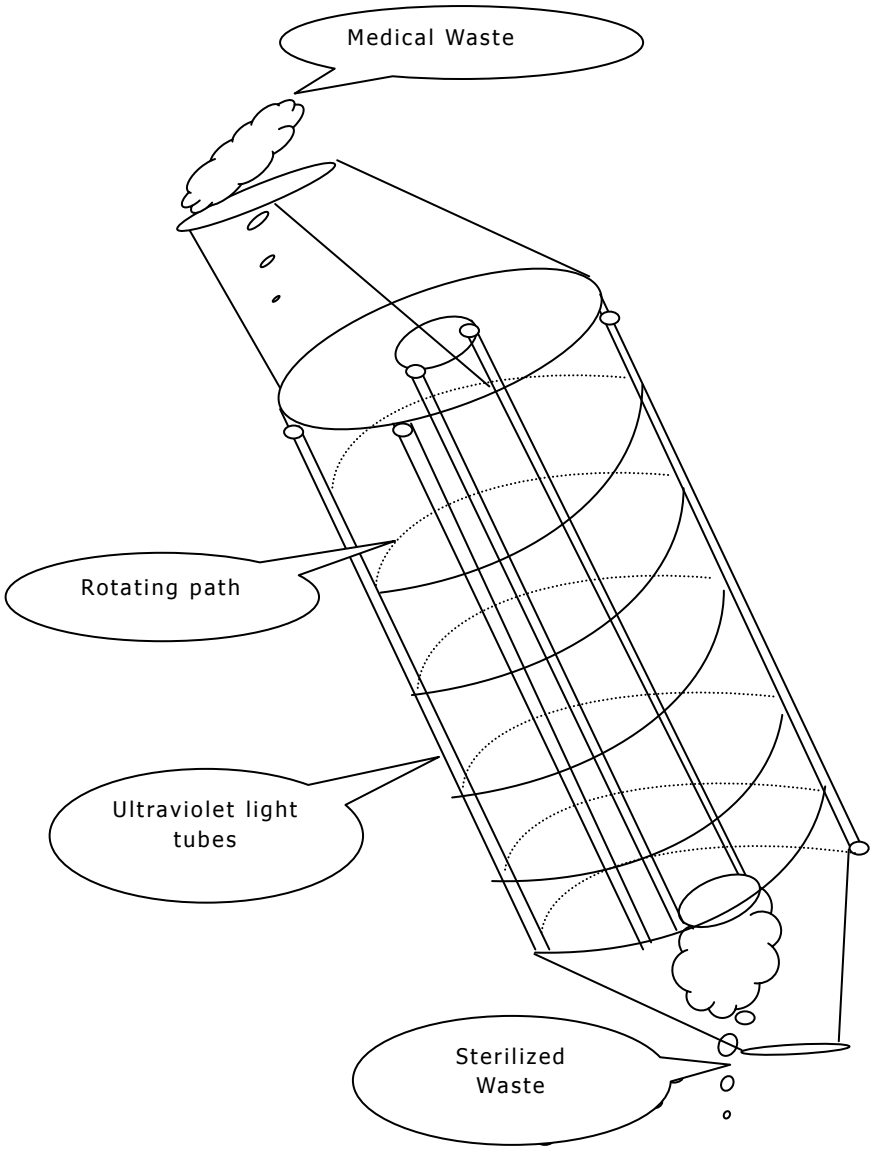


Figure 1: A rotating cylinderical device to sterilize medical waste through ultraviolet light as the waste follow a helical path to the exit.

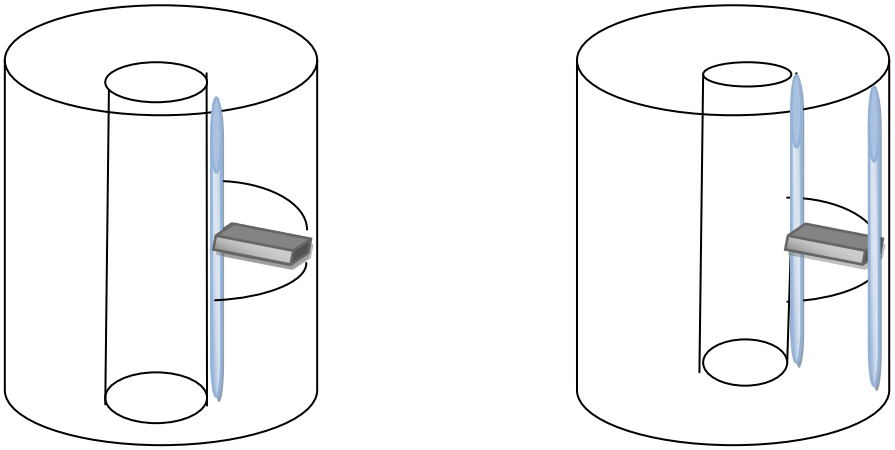
The path will have all that it needs to allow all the pieces move down simultaneously, without getting stuck somewhere along it's way down to exit. The medical waste will get exposed to ultraviolet light through the tubes along the devices. The tubes remain in their position, while the pieces of waste will rotate around them. Each piece of waste, after having certain exposure to light, will leave the device with hopefully the minimal viral or bacterial content left over.

The company would like to have some scientific guidance concerning the

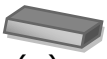
- number of Ultraviolet(UV) light tubes to be used
- location of the tubes
- exposure time, thus optimal rotational velocity of the device
- effect of having light sources of different magnitudes

which, all together, will hopefully help for a better design.

The study group considered only one dimensional(radial) variations between the inner and outer cylinder, as shown in Figure 2:



(a) (b)

Figure 2: A piece of medical waste  along a path between two cylinders, exposed to sigle (a) and double (b) UV light sources

Two cases are considered: a UV light source from one side(a) and two sides (b) A schematic of an object to be sterilized is shown in Figure 3.

$x = 0$ $x = d$

Figure 3: Schematic of an object to be sterilized with UV light sources at each end

Model assumptions:

- We assume that the object has a length d, which is also the distance between the inner and outer cylinder.
- The light source penetrates through the sample and decays exponentially as a function of the distance from the source: For a source located near $x = 0$, we assume that the sample gets exposed to a light source of the form
$$Q(x) = qe^{-kx}, q > 0, k > 0$$
from a light tube near $x = 0$,whereas the tube located near $x = d$,

would yield a source
$$Q(d - x)$$

- The rate of decay of medical content(concentration) $c(x,t)$ at point x and time t is proportional to the the existing concentration at that point, with the decay factor $Q(x)$, the strength of ultraviolet light acting on the object at that point.

2. Models and analysis

We consider three cases:

Case I: a single UV light source located along the inner or outer cylinder.

Case II: two light sources located along both inner and outer cylinder near both ends of the object.

Case III: As in case II, but with sources of different magnitudes

Case I Single light source

Therefore, for a single light source we have the simple model

$$\frac{\partial c(x,t)}{\partial t} = -Q(x)c(x,t) \tag{1}$$

with the initial condition

$$c(x,0) = 1 \tag{2}$$

meaning that the object is initially contaminated hundred percent. The solution to the Cauchy problem is

$$\begin{aligned} c(x,t) &= \exp(-Q(x)t) \\ &= e^{-(qe^{-kx})t} \end{aligned} \tag{3}$$

Remarks

- For each fixed $t, c(x,t)$ is an increasing function of x as

$$\frac{\partial c}{\partial x} = kqe^{-kx}e^{-(qe^{-kx})t} > 0$$

This means that more medical content will remain in the object as we examine it further away from the UV light source, which is expected. For $q = 1, k = 1$, and t=0,1,...,10 değerleri için mesh plot of $c(x,t)$ is given in Figure 4

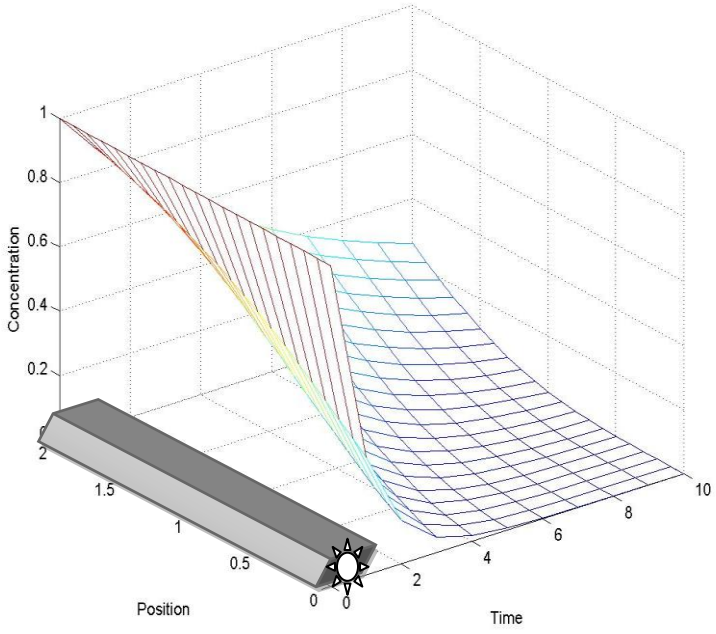


Figure 4: An object(rectangular prizm) under sterilization with a single light source as t increases

- For each value of x, the concentration decreases exponentially as a function of time, with the highest values along the other end, $x = d$.
- For a given $\epsilon > 0$, the time T_1 it takes to have the maximum concentration, the concentration at $x = d$, to be equal to ε can be determined as follows:

$$c(d, T_1) = e^{(-qe^{-kd})T_1} = \epsilon$$

from which we get

$$T_1 = \frac{e^{kd}}{q}\ln(\frac{1}{\epsilon})$$

So we have $c(x,t) < \epsilon$ for all $(x,t), 0 \le x \le d, t > T_1$.

Case II: Double light sources of the same magnitude

In this case the decay factor at a point x will be the sum of the light sources $Q(x)$ and $Q(d-x)$. Therefore, we will have

$$\frac{\partial c(x,t)}{\partial t} = -(Q(x) + Q(d-x))c(x,t) \tag{4}$$

with the same initial (2). The solution to (2)-(4) is

$$c(x,t) = e^{-(Q(x)+Q(d-x))t} \tag{5}$$

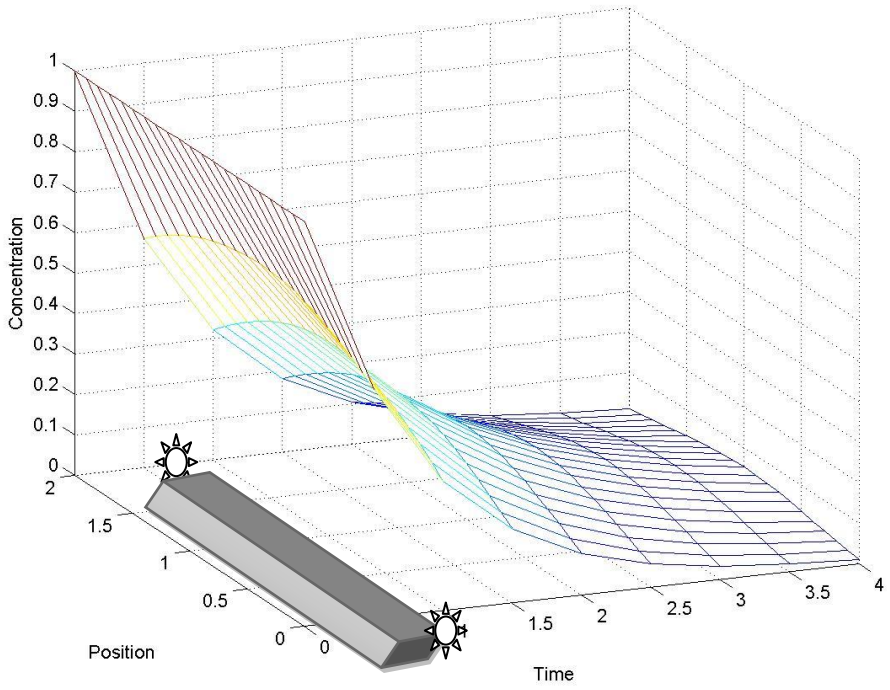


Figure 5: An object(rectangular prizm) under sterilization with two light sources as t increases

Remarks

- For each value of x, the concentration decreases exponentially as a function of time, with the highest values along the middle, $x = \frac{d}{2}$ where $Q(x) + Q(d-x)$ assumes its smallest value.

- For a given $\epsilon > 0$, the time T_2 it takes to have the maximum concentration, the concentration at $x = d/2$, to be equal to ε can be determined as follows:

$$c(d/2, T_2) = e^{-2(Q(d/2)T_2} = \epsilon$$

From which we get

$$T_2 = \frac{e^{kd/2}}{2q} \ln(\frac{1}{\epsilon})$$

So we have $c(x,t) < \epsilon$ for all $(x,t), 0 \le x \le d, t > T_2$.

- The comparison between T_1 and T_2:
we would like to compare the time values to have the maximum concentration become less than a prescribed value ϵ to see the effect of having UV light sources on each side of the cylinder.

$$T_2 = \frac{e^{kd/2}}{2q} \ln\left(\frac{1}{\epsilon}\right) = \frac{1}{2}\sqrt{\frac{\ln(\frac{1}{\epsilon})}{q}}\sqrt{\frac{e^{kd}}{q}\ln(\frac{1}{\epsilon})} = \frac{1}{2}\sqrt{\frac{\ln(\frac{1}{\epsilon})}{q}}\sqrt{T_1} \quad (6)$$

The relation (6) implies that sterilization time with two light sources is proportional to that required by a single light source. So, quite unlike the common sense expectation of the relation $T_2 = \frac{1}{2}T_1$, the advantage of having two sources is considerably high.

- The advantage of having two sources is apparent from the illustrations shown in Figure 4 and 5. The concentration decays slowly and ununiformly with a single light source as shown in Figure 4, whereas the decay is much faster and has a rather uniform nature with two sources of light, as shown in Figure 5.
- Average concentration versus time graphics are illustrated in Figure 6.

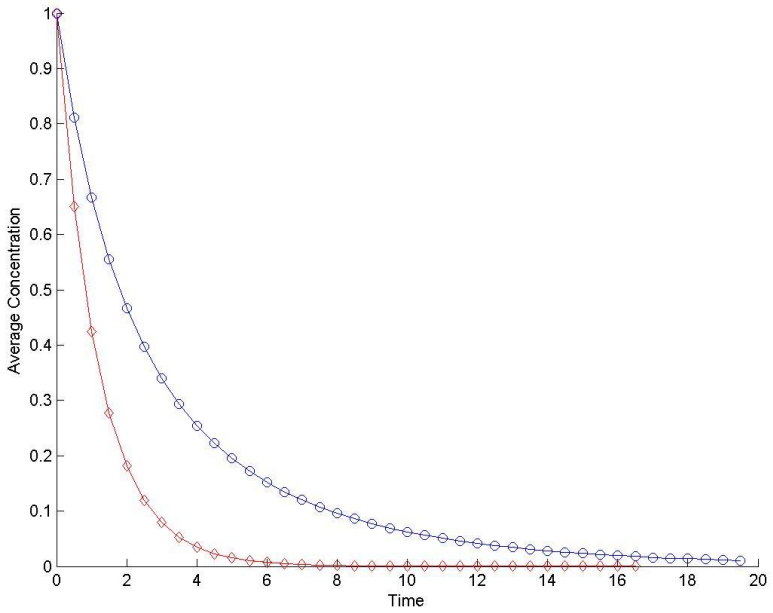


Figure 6: Average concentration versus time:(o) single, (◇red) two light sources

Case III: Double light sources of different magnitudes

In this case we assume that the two light sources with different magnitudes: $Q_1(x) = q_1 e^{-kx}, Q_2(d-x) = q_2 e^{-kx}$

So then we have

$$\frac{\partial c(x,t)}{\partial t} = -(Q_1(x) + Q_2(d-x))c(x,t) \tag{7}$$

with the same initial (2). The solution to (2),(7) is

$$c(x,t) = e^{-(Q_1(x)+Q_2(d-x))t} \tag{8}$$

The solution (8) have also the same qualitative behaviour as the previous ones, except that setting $\frac{\partial c}{\partial x} = 0$ we have $x = \frac{d}{2} + \frac{1}{2k}\ln(\frac{q_1}{q_2})$, where $c(x,t)$ assumes its maximum for each t. In this case we have an asymmetric sterilization along the sample. In case $q_1 = q_2$, the maximum point shifts to the center, expectedly.

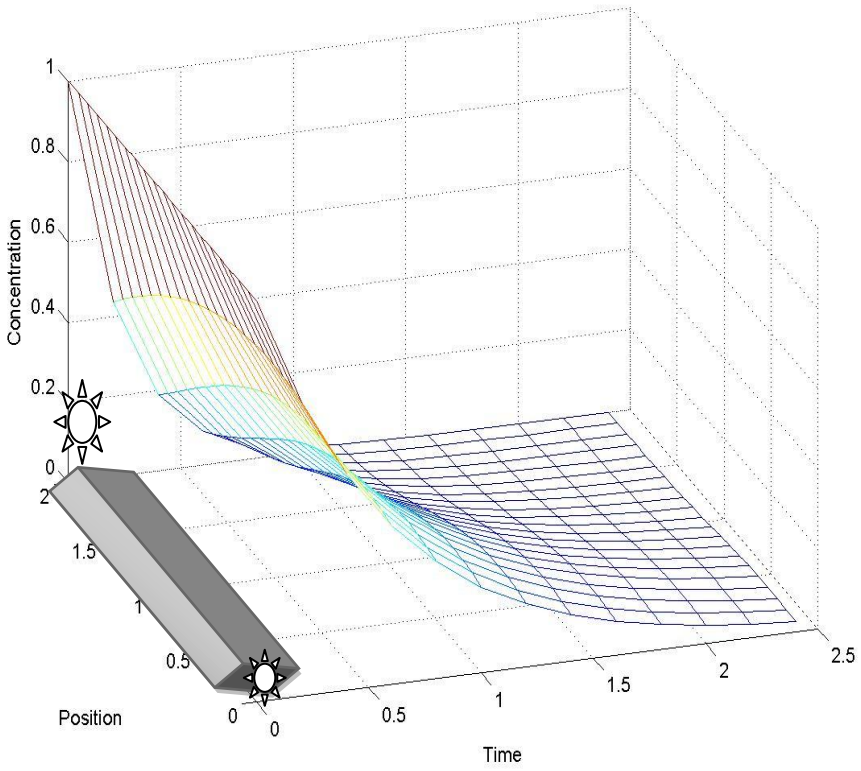


Figure 7: An object(rectangular prizm) under sterilization with two light sources of different magnitude($q_1 = 1, q_2 = 4$) as t increases

Finally we can determine the time it takes to have a concentration less than a prescribed value ϵ.

Similar to the algebra above, we find the required time to be equal to

$$\overline{T_2} = \frac{e^{kd/2}}{2\sqrt{q_1 q_2}} \ln(1/\epsilon) \tag{9}$$

Notice that if $q_1 = q_2$ then we have $T_2 = \overline{T_2}$.

There does not seem to be a practical advantage for using UV lights of different magnitude.

3. Optimal Rotational Velocity

Having determined the time needed to have a concentration less than a prescribed value for each cases, the next question is how to determine the rotational velocity of the device so as to keep the object within the sample for the estimated amount of time.

We will carry out the computations for each Case. But first we look at the first case

Let b the axial distance between the helical curves , L the length of the cylinderical device

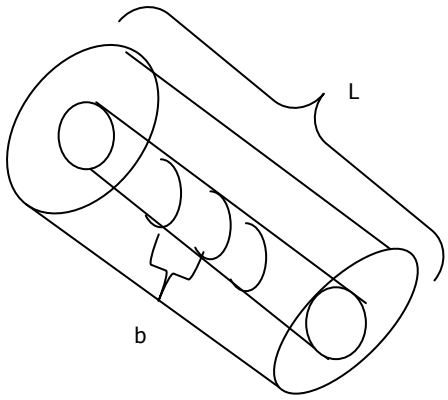


Figure 8: Helical curves around the inner cylinder with radius r

Let ω be the rotational speed of the inner cylinder and v be the downward velocity of the object on the helical path.

Then the time required for a single rotation is $\tau = 2\pi/w$. Since b units of axial distace is travelled during this time, the axial velocity of the object is given by $v = \frac{b}{\tau}$

The time it takes for the object to complete its travel is then given by $T = \frac{L}{v} = \frac{L\tau}{b} = \frac{2\pi L}{wb}$

which is also the object's residence time in the device. For the required level of sterilization with a single light source, $T \geq T_1$, i.e.,

$$\frac{2\pi L}{wb} \geq \frac{e^{kd}}{q}\ln(\frac{1}{\epsilon})$$

or, $w_1 = w$ will have to satisfy

$$w_1 \leq= \frac{2\pi L}{bT_1} = \frac{2\pi L}{\frac{e^{kd}}{q}\ln(\frac{1}{\epsilon})b}$$

Asuming that one needs to save energy, then optimal rotational speed will be given by

$$w_1(opt) = \frac{2\pi L}{\frac{e^{kd}}{q}\ln(\frac{1}{\epsilon})b} \tag{9}$$

Similarly for case II, we need $T \geq T_2$, so we have

$$w_2 \leq= \frac{2\pi L}{bT_2} = \frac{2\pi L}{\frac{e^{kd/2}}{2q}\ln(\frac{1}{\epsilon})b}$$

Therefore, the optimal rotational speed will be given by

$$w_2(opt) = \frac{2\pi L}{\frac{e^{kd/2}}{2q}\ln(\frac{1}{\epsilon})b} \tag{10}$$

Comparing $w_1(opt)$ and $w_2(opt)$ using the relation

$$\frac{w_1(opt)}{w_2(opt)} = \frac{T_2}{T_1} = \frac{\frac{1}{2}\left(\frac{\ln\left(\frac{1}{\epsilon}\right)}{q}\right)^{\frac{1}{2}}}{\sqrt{T_1}}$$

Or

$$w_2(\text{opt}) = \frac{w_1(\text{opt})\sqrt{T_1}}{\frac{1}{2}\left(\frac{\ln\left(\frac{1}{\epsilon}\right)}{q}\right)^{\frac{1}{2}}}$$

4. Conclusions and recommendations

The simplest as is, the the model reveals many important clues for an effective design of a medical sterilizer.

- We determined the time needed to sterlize a single object up to a given required precision by using single and double light sources along the inner and outer cylinders.
- We determined that the time needed to sterilize an object with light sources along both inner and outer tubes is proportional to the square root of that required by a single light tube. This result, though seem to contradict with common sense, implies considerable saving .
- We determined optimal rotational speeds that will allow the object to remain in the system so a to get sterilized to a give precision. Furthermore, developed a relation between optimal rotational speeds of the device having a single or double light tubes. Adding an extra light tube allowes for a faster rotation.
- Light sources of different magnitudes do not lead to any advantage in terms of sterlization time, other than just the shift in the location of the path for the maximum UV exposure.
- The parameters q, k in the decay factor $Q(x) = qe^{-kx}$ can be estimated for certant type of medical wastes, this, in turn, will effect rotational speed.
- The model assumes a continuous exposure from the light sources, whereas in reality, the object's distance to light source changes as it revolves. So a more realistic model should be developed to take this situation into account.

5. Acknowledgements

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References

[1] Wladyslaw Kowalski, Ultraviolet Germicidal Irradiation Handbook, Springer-Verlag, 2009.