Chapter 6

Non-monotonic variation of stress intensity with flaw size in metal-lined fibre-reinforced pressure vessels

Dr A. Akhtar,Director Materials Engineering
Powertech Labs Inc, Surrey, Canada
Report by Rex Westbrook
Math and Stat, University of Calgary

1 Problem description

Metal-lined continuous fibre reinforced plastic (FRP) over-wrapped pressure vessels are used in aerospace applications, for storage of breathing air in fire fighting and scuba diving and for the storage of compressed gaseous fuels on natural gas and hydrogen vehicles. Continuous fibres, generally about 15 μm in diameter and made of glass, carbon or kevlar, are embedded in a polymer matrix. The metallic liner is made of a ductile material - generally steel or aluminum alloy

After the metallic liner has been wrapped, the composite vessel is subjected to a process termed Autofrettage. In that process the vessel is internally pressurized to the point that the ductile metal liner undergoes a small amount of plastic deformation (unlike elastic deformation which disappears upon removal of stress, plastic deformation remains after the stress has been removed). Upon depressurization of the vessel, the metallic liner remains under compression and the FRP under tension.

Acoustic emissions associated with fiber breakage are being developed currently as a non-destuctive means of assessing the structural integrity of metal-lined continuous FRP over-wrapped vessels. Laboratory experiments have been carried out with flaws such as cracks and saw cuts of varying dimensions oriented in an axial-radial plane and located in the metallic liner, in the FRP or in both. Pressurization of the flawed vessel leads to fiber breakage, the extent of which is being examined with the intent that it will be a measure of structural integrity of the vessel. The results, however, suggest that the acoustic emissions attain a maximum for an intermediate flaw size. Low emissions are recorded when on the one hand the vessel has insignificant flaws, or on the other if the vessel has serious flaws. This non-monotonic variation of acoustic emission occurs whether the flaws are located in the metallic liner, in the FRP or in both.

The experiments suggest that the stress intensity at the discontinuity (crack tip) attains a maximum at an intermediate flaw size. A mathematical corroboration is desired.

2 Model

Dr. Ahktar informed us that an axial cut in the FRP resulted in the removal of the "*hoop stress*" provided by the cut fibres leaving a problem in which the liner has a circular strip free of outer stress. This strip will then bulge. At this point the problem was dubbed, "The Hernia Problem."

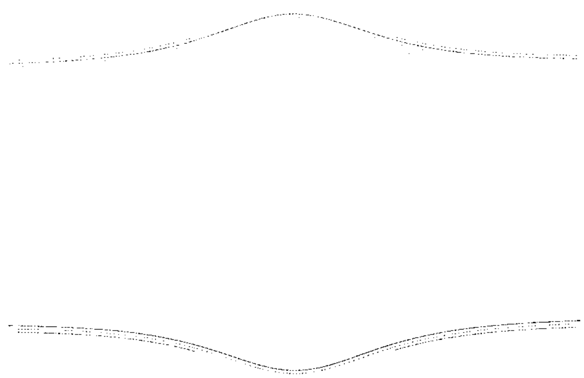

Figure 1: The Hernia Problem

The problem was modelled as an infinite elastic cylinder encased in fibres except for a strip of axial length l. It was found that the non-monotinicity could be explained by a Linear Elastic Shell model for the liner in which the FRP provided a normal force on the part of the shell that was wrapped because of the tension in the fibre. The fibres at the edge of the cut are stretched more than those further away.

The acoustic energy measured is due to the breaking of the fibres at the edge of the cut and its magnitude depends on the number of fibres breaking. This depends on the tension in the fibres and hence on the radial displacement of the liner at the edge of the cut.

In what follows we show that this displacement is a non-monotonic function of the crack length.

3 The Elastic Problem

Variables

E_s - elastic constant for the liner.

E_f - elastic constant for the FRP.

ν - Posson's ratio for the liner.

h_s - thickness of liner.

h_f - thickness of FRP.

a - radius of liner.

l - crack semi length.

w - axial displacement of liner (and FRP) from reference state.

p_0 - internal pressure on liner.

We define two constants $\alpha = p_0 a^2 / E_s h_s$, which has dimensions of length, and $\lambda = 1 + E_f h_f / E_s h_s$ which is dimensionless.

The liner is treated as an infinite circular cylindrical shell with the axial distance x measured from the centre of the bare strip caused by the cut. The problem is axially symmetric and the normal displacement w is a function of x only.

The fibres are considered as bands of elastic strings. The strain in the fibre band is w/a and the tension is $E_f w/a$. This produces a normal force $E_f h_f w/a^2$ on the outer surface of the shell. The equations for an elastic shell may be found in Timoshenko and Woinowsky-Krieger, Theory of Plates and Shells. After simple manipulation, they give the following differential equation for w.

$$\begin{aligned} \frac{h_s^2}{12(1-\nu^2)} \frac{d^4 w}{dx^4} + \frac{w}{a^2} &= \frac{\alpha}{a^2} \quad , 0 < x < l \\ \frac{h_s^2}{12(1-\nu^2)} \frac{d^4 w}{dx^4} + \frac{\lambda w}{a^2} &= \frac{\alpha}{a^2} \quad , l < x \end{aligned}$$

In deriving this equation, the axial stress resultant,which is constant,has been taken to be zero. The effect of a non zero constant is to change the value of α. Since the equations are linear, this will only lead to a change in the magnitude of w and will not change the nonmonotonic behaviour.

The boundary conditions for the problem are $w'(0) = w'''(0) = 0$, because of the symmetry; and as $x \longrightarrow \infty$, $w \longrightarrow \frac{p_0 a^2}{(E_s h_s + E_f h_f)} = \frac{\alpha}{\lambda}$, which is the normal displacement for the completely wrapped cylinder.

The usual length scale for a circular cylinder is $\sqrt{h_s a}$ and we define a length scale b for x as $b = \left(3(1-\nu^2)h_s^2 a^2\right)^{\frac{1}{4}}$. This leads to the following problem for w with " x " now being x/b and $L = l/b$.

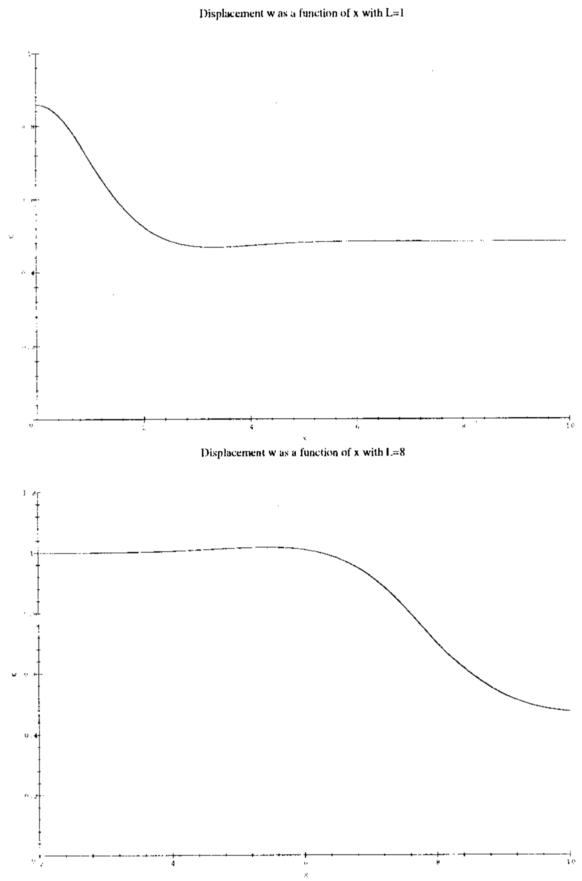

Figure 2: Plot of displacement w vs. x for two choices of L

$$w''''(x) + 4w(x) = 4\alpha \ , \ 0 < x < L,$$
$$w''''(x) + 4\lambda w(x) = 4\alpha \ , \ L < x,$$
$$w \longrightarrow \frac{\alpha}{\lambda} \text{ as } x \longrightarrow \infty,$$
$$w'(0) = w'''(0) = 0,$$
$$w, w', w'', w''' \text{ continuous at } x = L.$$

This problem is easily solved analytically and the displacement w can be obtained as a function of x (see Figure 2).

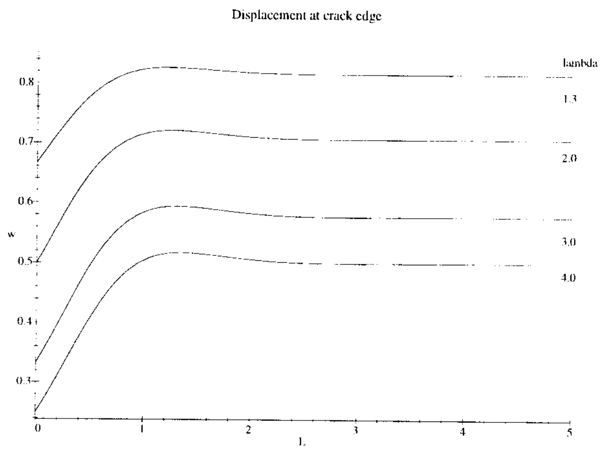

Figure 3: Plots of w vs. L for various λ.

Putting $x = L$ gives the displacement at the edge of the crack as a function of L. This is found to be non-monotonic. Graphs of w as a function of L are given in Figure 3 for some values of λ. It may be noted that the value of L at which w is a maximum increases as λ increases provided that b ,which depends on h_s and a remains constant. In the graphs, a unit on the L axis corresponds to a crack of length $2b$.

Two numerical examples were examined. The first with a steel liner had $E_s = 203,000$ MPa, $E_f = 43,700$ MPa, $h_s = 6.3$ mm, $h_f = 7.8$ mm, $a = 163.5$ mm, and $\nu = 0.28$. This gives $\lambda = 1.2665$ and $b = 41.4$ mm. The maximum displacement w occurs at $L = 1.2076$ or for a crack of length $2bL = 100$ mm. This is in reasonable agreement with Dr Akhtar's experimental result of about 3 in or 76.2 mm.

The second example with an aluminum alloy liner had $E_s = 71,708$ MPa, $E_f = 43,700$ MPa, $h_s = 13.8$ mm, $h_f = 6.3$ mm, $a = 152$ mm, $\nu = 0.3$. This gives $\lambda = 1.2782$ and $b = 58.87$ mm. In this case the maximum of w occurs when $L = 1.2087$ or a crack of length $2bL = 142$ mm. This time there is very good agreement with the experimental value of about 5.5 in or 139.7 mm.

From the Figure 3, it is seen that the value of L at which the maximum displacement occurs changes very little as λ changes and that a value of about $L = 1.2$ seems appropriate for a wide range of values for λ. The actual length of the crack for maximum displacement is then approximately $2.4b$ and is governed almost entirely by the thickness and diameter of the liner.

4 Appendix

We give here the solution of the differential equations and boudary conditions in a form which is convenient for the matching at $x = l$.

$$
\begin{aligned}
w &= A\cosh x \cos x + B \sinh x \sin x + \alpha, \qquad 0 < x < L \\
w &= e^{-\beta(x-L)}(a\cos\beta(x-L) + b\sin\beta(x-L)) + \frac{\alpha}{\lambda}, \qquad L < x, \qquad \beta^4 = \lambda
\end{aligned}
$$

This solution satisfies the boundary conditions at $x = 0$, and as $x \longrightarrow \infty$. To make the solution C^3 at $x = l$, we must solve the equations

$$
\begin{bmatrix} Cc & Ss & -1 & 0 \\ Sc - Cs & Sc + Cs & \beta & \beta \\ -Ss & Cc & 0 & -\beta^2 \\ -(Sc + Cs) & Sc - Cs & -\beta^3 & \beta^3 \end{bmatrix} \begin{bmatrix} A \\ B \\ a \\ b \end{bmatrix} = \begin{bmatrix} -\alpha(1 - \frac{1}{\lambda}) \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

where $C = \cosh L$, $S = \sinh L$, $c = \cos L$, $s = \sin L$. This was solved using Maple to give a as a function of L. The displacement at $x = L$ is $a + \frac{\alpha}{\lambda}$.