OXFORD STUDY GROUPS WITH INDUSTRY 1968-1971

PROGRESS REPORT ON
APPLICATIONS OF DIFFERENTIAL EQUATIONS
MATHEMATICAL INSTITUTE, OXFORD.

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Introduction

The initial proposal supported by the Science Research Council was for a small group of Oxford University faculty members based on the Mathematical Institute, Oxford, to try and make contact with research workers in industry who were concerned with problems in some way involving the theory, formulation, application or numerical solution of differential equations. The initial period was for three years and the methods of approach to industry were varied in the light of experience. The original motivation, philosophy and organisation of the Study Groups is described in an article written by L. Fox and A. B. Tayler in the December 1969 issue of the Bulletin of the Institute of Mathematics and its Applications, and will not be repeated here. The scheme has had some success and it would seem sensible at the end of the experimental three year period before commencing a further five year period, also supported by the Science Research Council, to review the contacts made and attempt to evaluate the effort involved.

Four one week Study Group meetings were held (one was postponed and one cancelled) and several follow up meetings took place. A simple description is to briefly discuss each problem chronologically and we do this in the next section, listing all the contacts made alphabetically by organisation in Appendix 1. Faculty members who have taken part in the meetings are listed in Appendix 2 and a typical Study Group programme is reproduced as Appendix 3. In the final section we try to analyse our successes and failures and make some tentative conclusions about the direction of future efforts.

The research problems

The first meeting was held in September 1968 and there were four participants from industry.

Mr Herne from the National Coal Board, Harrow, came to discuss the problem of the analysis of moving granular material and, in particular, the motion of large quantities of coal in a bunker. The existing models and

numerical methods were critically examined and found to be unsatisfactory. A further one day meeting was organised with 25 participants to attempt to construct a more realistic model for granular flow, and there were several meetings with Professor Spencer of Nottingham University and Dr Cheng of Warren Spring Research Laboratory. No really satisfactory model has yet been proposed and there is clearly a great need for mathematical work in this field especially with regard to the properties of powders.

Dr Huggill, from ICI Wilmslow, had a problem concerning the injection moulding of a circular disc of plastic. The model was well established but direct computing times were excessive. A partially analytic approach was discussed and developed at the meeting together with suggestions for improving the numerical techniques used. No further contact has been made with Dr Huggill although there are several interesting unanswered questions which arise out of other aspects of the process which could possibly provide material for a student's thesis.

Mr Paull, from Rolls Royce Bristol, was concerned with viscous flows in situations such as internal flows where standard boundary layer computations were inappropriate. He proposed a model for such a situation and its qualitative properties were obtained at the meeting. Considerable doubt was cast on the model in the discussion and no further contact has been made.

Dr Robertson, from ICI Wilmslow, was concerned with the numerical solution of strongly coupled non-linear ordinary differential equations (stiff system). Considerable discussion took place at the meeting, and afterwards, about the best methods for dealing with such systems.

Mr Brereton from CEGB, Computing Branch had a problem concerning the dynamics of steam generation in a nuclear-fired once-through boiler. The model used consisted of the usual one dimensional compressible flow equations for the three phases, namely water, wet and dry steam. The unknown position of the interfaces between the phases gives rise to a difficult free-boundary computation. Since the primary physical interest was in the thermal behaviour, which has a much longer time scale than the dynamic behaviour, a simpler model was proposed which Mr Brereton agreed to test numerically. The results of this computation have not been communicated back to Oxford.

Mr Cardwell from British Railways, Derby was concerned with the dynamic behaviour of a current collection system for all electric locomotive. After considerable discussion at the meeting, and afterwards, a much simpler model was constructed and its solution both analytically and numerically led

to some interesting and novel mathematics. The results have been published in Ockendon & Tayler, Proc. Roy. Soc. A 322 and Fox, Mayers, Ockendon & Tayler, J. Inst. Math. App. 8 (1971).

Dr Hill from UKAEA, Aldermaston asked for a detailed discussion of the numerical solution of the Boltzmann transport equation. A review of existing methods was given and suggestions made about using methods which exhibit conservation in a global sense. Dr Hill agreed to try and assess the effectiveness of these procedures.

Mr McGill from Yarrow & Co. was concerned with the response of a ship shafting system to an imposed shock or rapid transverse displacement at one end. Numerical procedures using the full Timoshenko beam equation had encountered serious instabilities. A perturbation analysis proved possible and a complete qualitative picture of the various response regimes was obtained. With this information a computation of any given regime would be quite straightforward but Mr McGill had in the interim abandoned the project so no numerical work was carried out.

Mr Marks from N.C.B., Cheltenham had a problem concerning the combustion of coal in a fluidised bed of ash. He proposed a model in terms of difference equations, which was reformulated in terms of a hyperbolic system of differential equations. This model was developed by a graduate student for a Diploma (now M.Sc.) thesis but only partially explains the phenomena. An improved version which correctly models the scattering of the coal when the fluidised bubble bursts at the surface of the ash is required, but has not yet been formulated.

Dr Scriven from CEGB, Leatherhead had effectively solved his problem of smoke dispersion from a power station chimney. His model consisted of a diffusion equation for the smoke concentration with coefficients depending on height and he was looking for upper bounds for the solution at ground level. Possible ways of obtaining rigorous bounds analytically were discussed and Dr Scriven agreed to investigate them further.

Mr Caress from Pilkingtons, Ormskirk was concerned with the rolling of molten glass between two large water cooled rollers. The flow is essentially two dimensional and upstream of the rollers an almost circular 'bolster' is formed. The problem is to determine the temperature distribution and so arrange conditions that high temperature gradients are avoided. A considerable amount of time and effort has been expended on this problem both analytically and numerically in Oxford but no useful results have as yet been obtained.

Mr Clarke from BSRA, Wallsend had a problem concerning the stability of the steered motion of a ship when using an auto-pilot. His proposed model was essentially a fifth order non linear system of ordinary differential equations containing three parameters which have to be optimised. It was not at all clear in what sense this optimisation should be effected and several procedures were proposed to Mr Clarke. There has been some contact since the meeting and further problems have also been discussed.

Mr Hobbs from ICI, Billingham had a model for the pollution of the river Tees by industrial effluent. This model required an elaborate computation which was proceeding successfully except for certain difficulties with boundary conditions. Suggestions were made for simple changes of variable which avoided these difficulties and resulted in a well posed problem.

Mr Hopkinson from UKAEA Winfrith was concerned with the solution of a model for a shell and tube counter-flow heat exchanger. A numerical method was proposed in detail and it was left to Mr Hopkinson to carry it out.

Mr Rothman from BICC, London brought the problem of moisture transport in layered paper systems, as encountered in the drying process for insulated cables. Considerable discussion took place about what features a suitable model should contain and no agreement was reached. The faculty participants wanted more experimental data on the macroscopic properties of layered paper before having any confidence in the model proposed by Dr Rothman.

Mr Rothwell from B.P., Epsom, was concerned with the rate of polymerisation of a hydrocarbon in the presence of an inhibitor. His model was a third order system of ordinary differential equations in which exponential non linear terms appear. Unsatisfactory results had been obtained using a standard computing programme. An improved programme was run in Oxford which gave satisfactory results. A CAPS award has now been given to a graduate student to work jointly with the Oxford Computing Laboratory and B.P. Epsom on problems of this kind.

Mr Jenkins from Pilkingtons, St Helens brought the problem of the flow of molten glass in a forehearth and the consequent temperature distribution. Other geometries were also considered and some progress made for special values of the quantities concerned. A graduate student is currently considering a wide range of variable viscosity flows with viscosity exponentially dependent on the temperature, and has written his M.Sc. thesis on this topic. There is also consultation with the Salford Fluid Mechanics

Computation Centre who have some experience of steady flows of molten glass under gravity.

Dr Morton from CEGB, Leatherhead was concerned with the impact of a spherical water droplet on a wet horizontal surface and the resultant splashing. This is a difficult and complicated problem and a detailed critique of current methods and assumptions was given. Further analytical and numerical work is proposed, the former on the similarity problem of a wedge of water striking a semi-infinite region of water. A graduate student has written an M.Sc. thesis on this topic.

Mr Reagan from ICI, Billingham was concerned with temperature distributions in furnaces. Simple one dimensional models have been satisfactorily solved but the extension to two dimensions is extremely difficult. A numerical procedure was proposed which is to be investigated further in Oxford if a suitable graduate student can be found. The relevance of a simple diffusion model was also discussed and is to be investigated further.

Mr Tough and Dr Leigh from BISRA, London were concerned with the rolling of cold steel and wanted to determine the stress distribution in the strip due to wear on the rollers. Present models are all one dimensional and attempts were made to extend them to two dimensions. This was done for the strip but no satisfactory three dimensional model has as yet been formulated for the plastic region between the rollers.

Other contacts

Some companies have asked representatives of the Study Groups to visit them rather than one of their employees coming to Oxford for a week. Several visits have been made to Morris-Leyland in Oxford and two areas of problems have been discussed at length. The first concerns car heating systems and simple models are being developed for both the heating of the interior and the heating of the windscreen. A graduate student discussed analytical solutions of the former in his diploma (now M.Sc.) thesis, and further experiments are in progress to test these results. The windscreen problem is more difficult and a satisfactory solution has not been obtained. Further experimental work on a related problem of the design of cooling jets in a deep freeze cabinet is being carried out in the Oxford Engineering Department. The second area of problems concerns an analysis of welding,

that is heat conduction problems with variable density heat sources and a phase change boundary with latent heat. A semi-analytical method is being developed to deal with a simple one dimensional configuration by a graduate student for his Ph.D. thesis.

A similar mathematical problem concerning the casting of rods was suggested by a visit to Alcan Aluminium, Banbury, and work is in progress on this problem. A visit was also made to Serck Ltd., Birmingham where the design of heat exchangers and taps was discussed but no further action resulted.

Evaluation of the scheme

There is some concrete achievement to be shown for the efforts of those taking part. Briefly our successes are:

(a) We have made substantial contributions to half a dozen problems and some contribution to a further dozen.

(b) Our graduate students have benefited enormously from the meetings and the faculty members have found interesting problems.

(c) Almost without exception our visitors have said they found the week in Oxford very stimulating and most of them wished to remain in contact.

(d) We have learnt a great deal about industrial attitudes and our visitors have seen some of the problems of educating graduate students in research.

Our failures are:

(a) The response to publicity in scientific magazines and a mailing list of 120 organisations has been very sparse. We have only just obtained sufficient response to justify each meeting and did indeed cancel one. Hence the problems discussed have not been as interrelated as we planned and in general we have only made contact with very big organisations.

(b) It has been very difficult to remain in contact with our visitors and most industries seem to have little long term commitment to any particular problem. They do not appear to accept that the very fundamental nature of mathematical research implies that no immediate solution to their problem is likely to be available.

In terms of our original ideas we did not expect the Study Groups to be so valuable and interesting for graduate students. This now becomes one of the

major features of the scheme and the non-technical interaction has been very stimulating. Nor did we expect the response from industry to be so difficult to elicit and clearly we must carefully think out our policy for obtaining the first contact very carefully. For the next five years possible improvements in this situation should be:

(a) The existence of a full time secretary and organiser, Dr J. Ockendon, who should be able to mount a much more efficient publicity machine and have more time for visits. The personal approach through the Department of Trade & Industry and the C.B.I. seems a fruitful avenue to be explored.

(b) The encouragement of cooperation with engineering and chemistry departments who are engaged in work relevant to industrial problems and who need theoretical advice. Thus we should in addition try to organise a two stage industrial cooperation scheme with the experimental faculty member as the middle man. This will require considerable publicity in applied science departments in all universities and is in line with current S.R.C. thinking about more cooperation between faculties.

(c) The Development of the CAPS awards scheme for graduates reading for research degrees at Oxford. We should request more flexibility in the conditions for the award of SRC grants for Engineering Mathematics and have more publicity about the way these grants can be used at Oxford.

(d) An attempt to interest industry in a scheme whereby their senior research and development workers could come to Oxford for a 'sabbatical' term to learn about new developments in applied mathematics, numerical analysis and computing, under the guidance of a faculty supervisor.

A.B. Tayler.
December, 1971.

Appendix 1

Industrial and Government participant organisations

ALCAN Research and Development Ltd., Banbury.
British Insulated Callender's Cables Ltd., Central Research and Engineering Div., London, W.12.
British Leyland, Pressed Steel Fisher Research Labs., Oxford.
British Iron and Steel Research Assn., Control Systems Section, London, S.W.11.
British Petroleum, Chemicals Ltd., Research & Development Dept., Epsom.
British Railways Board, Railway Technical Centre, Derby.
British Ship Research Association, Wallsend, Northumberland.
Central Electricity Generating Board, Computing Branch, London, S.E.1.
Central Electricity Research Laboratories, Leatherhead.
Imperial Chemical Industries, Agricultural Division, Billingham.
Imperial Chemical Industries, Heavy Organic Chemicals, Billingham.
Imperial Chemical Industries, Management Services Division, Wilmslow.
Ministry of Technology, Warren Spring Lab., Stevenage.
National Coal Board, Operational Research Branch, Harrow.
National Coal Board, Research & Development Dept., Cheltenham.
Pilkington Bros., Control Systems Dept., St Helens.
Pilkington Bros., Research & Development Labs., Ormskirk.
Rolls-Royce Ltd., Propulsion Dept., Bristol Engines Divison.
United Kingdom Atomic Energy Authority, Control & Instrumentation Division, Winfrith.
United Kingdom Atomic Energy Authority, Atomic Weapons Research Est., Aldermaston.
Y-ARD Consultants, Yarrow & Co., Glasgow.

Appendix 2

Faculty Participants in Study Groups

Faculty

Dr A Day
Dr D. Donnelly
Professor L. Fox
Dr I. P. Grant
Professor A. E. Green
Dr J. T. Lewis
Dr J. B. McLeod
Dr D. F. Mayers
Dr. J. D. Murray
Dr H. Ockendon
Dr J. R. Ockendon
Dr M. E. Rayner
Dr. D. A. Spence
Dr. A. B. Tayler
Professor L. C. Woods

Professor S. Antman (on leave from New York)
Dr P. Christophers (Cranfield)
Professor M. Glauert (East Anglia)
Dr Rasmussen (Southampton)
Dr J. Walsh (Manchester)

together with approximately twenty-five graduate students.

Appendix 3

Programme for the Fourth Oxford Study Group, 15th-19th March, 1971

	Speaker	*Lecture*
Monday, 15th March		
9.30	Mr Morton	C.E.G.B. Problem
11.15	Mr Reagan	I.C.I. Problem
2.30	Dr Tough and Dr Leigh	B.I.S.R.A. Problem
4.15	Mr Jenkins	Pilkingtons Problem
Tuesday, 16th March		
9.30	Professor L. Fox	Ill conditioned numerical problems and instabilities of numerical methods.
11.15	Discussion Groups	
2.30	Dr J.D. Murray	Mathematical Modelling
4.15	Discussion Groups	
Wednesday, 17th March		
9.30	Dr A.B. Tayler	The use of analytical methods before extensive computation
11.15	Discussion Groups	
2.30	Dr D.F. Mayers and Miss J. Taylor	Numerical methods involving free boundary problems
4.15	Discussion Groups	
Thursday, 18th March		
9.30	Dr J.R. Ockendon and Dr. H. Ockendon	Wave propagation and partial differential equations
11.15	Free	
2.30	Dr Donnelly and Dr Grant	Numerical methods for partial differential equations
4.15	Free	
Friday, 19th March		
9.15	Discussion of B.I.S.R.A. Problem	
10.00	Discussion of I.C.I. Problem	
11.15	Discussion of C.E.G.B. Problem	
12.00	Discussion of Pilkington Problem	
2.30	Further discussion time and review of Study Group organisation.	

Coffee will be taken between 10.45 and 11.15 and lunch will be sandwiches at the Nag's Head, Hythe Bridge Street at about 1 p.m. Tea will be between 3.45 and 4.15. Dinner is 7 p.m. for 7.15 in St Catherine's College Senior Common Room. Rooms available in the Mathematical Institute are the Titchmarsh Room (coffee and tea), L3 (formal sessions), SR1, 2 and G9 for working desks and discussion groups.

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1974-1977

THIRD PROGRESS REPORT ON
APPLICATIONS OF DIFFERENTIAL EQUATIONS
MATHEMATICAL INSTITUTE, OXFORD

1. Introduction

This report describes the activities of the Oxford Study Groups with Industry during their third three year period of operation. With continuing support from the Science Research Council and more recently from Oxford University, ten meetings have now been held and the number of problems considered exceeds one hundred. The two basic aims of helping research workers in industry and enabling academic applied mathematicians to work on practical problems have remained unchanged throughout.

Following the pattern of previous reports the new experience gained over the past three years will be reviewed here after a chronological listing of the problems submitted during that time. More detailed information about any of the problems can be obtained from the Study Group Secretary, Dr. J. Ockendon, Mathematical Institute, 24-29 St. Giles, Oxford.

2. Problems discussed at Study Groups

Dr. Ackroyd from UKAEA Risley proposed a finite element procedure for determining the even parity flux density in a neutron transport problem. Little was done at the Study Group other than to reassure Dr. Ackroyd of the promise held out by his method, which has subsequently proved highly successful in practice.

Mr. Anglesea from Babcock & Wilcox reviewed progress which had been made on the prediction of temperature distributions in two phase steam-air flow in boilers, this problem having already been mentioned in the 1971-1974 report. Linear and weakly nonlinear stability analyses have now been carried out by a research student in Oxford; in particular this work has shed new light on the stability implications of the response curve of mass flow as a function of pressure drop.

Dr. Blackburn from Parsons was also interested in the flow of a steam-air mixture but from the viewpoint of condenser design. Some suggestions were made at the Study Group concerning the numerical solution of the pair of coupled nonlinear elliptic equations to which Dr. Blackburn's model reduced in a simple situation but the design problem in which the total condenser pressure appears as an eigenvalue raised novel analytical questions which have still not been considered in detail.

Mr. Booth from B.P. brought the problem of the vapourisation of a liquid in contact with a hot solid and this situation was modelled at the Study Group as a moving boundary problem involving both conduction and convection. An analytical solution was found for small times which in principle permitted a start to be made on a numerical computation, but this has yet to be carried out.

Dr. Buchanan from NCB posed the intriguing problem of fault location in a coal seam through reflection of a seismic wave generated at the coal face. A combined numerical and analytical approach was suggested which complemented Dr. Buchanan's transform solution. The application of the ray method proved to be especially advantageous for variable width coal seams.

Dr. Ewing from CERL was interested in the effects of small concentrations of gas bubbles on pressure surges in water filled pipes and there was considerable discussion at the Study Group about which was the most important damping mechanism. Attention was drawn to the work being done at Enschede and to related literature in relaxing gas flow. There has since been considerable correspondence on the effects of slip and heat transfer and the problem has proved of sufficient mutual interest for the allocation of a CASE award.

Mr. Massey and Miss Boxall from B.S.C. Grangetown were concerned with reactive mass transfer in a porous medium. Uncertainty about the diffusion coefficients and heats of reaction was a deterrent to either analytical or numerical progress at the Study Group although the full equations seemed amenable to the Crank-Nicolson method. The Study Group is now much better prepared to deal with this type of problem as a result of work on Mr. Moore's problem (q.v.).

Mr. Hoad from ICI Mond returned with a modification of his 1974 problem of minimizing the energy loss in conducting electric current

between two plates joined by an array of spigots. His revised geometry permitted an explicit solution by conformal mapping in one particular limiting case. The usefulness of the conjugate gradient method in computing the potential in the original version of this problem has recently been assessed by a D.Phil. student in Oxford.

Messrs. Pullum and Beasley from Standard Telecommunications brought the first probability problem to be considered at the Study Group. This was to determine the distribution of a combination of variates from a knowledge of their individual distributions but, apart from providing a few references, the discussion at the Study Group was too general to be of value in the specific examples of interest.

Mr. Booth from BP returned in 1976 with the fascinating problem of absorption of water wave energy by floating visco-elastic mats. During the Study Group, the problem was formulated as an integro-differential equation for the displacement of the mat, whose bending stiffness was such that regions of rapid variation and possibly greatest energy absorption occurred at the edges of the mat. A preliminary numerical calculation of the first few modes in an eigenfunction expansion has been carried out and a Weiner-Hopf analysis for a semi-infinite mat is being investigated as the next of several possible approaches to this problem.

Dr. Green from CEGB, Marchwood was concerned with the numerical solution of a one-dimensional model for the flow of steam and water in a boiler when large enough pressure changes could occur for the flow to reverse. His hyperbolic system of four first order partial differential equations did not permit any analytical simplification and only comments of a general numerical nature were made at the Study Group.

Messrs. Halliday and Sefton from BAC posed a problem arising in the control system for aircraft tailplanes which required the knowledge of the periodic solution of a set of eleven nonlinear ordinary differential equations. Little progress was made at the Study Group with finding a sufficiently efficient method of directly integrating the initial value problem out to the periodic solution, but eventually a one-dimensional model of a frictional oscillation under the action of a long period forcing term was proposed. This both permitted an approximate analytical description and gave some insight into the original problem. Of particular interest was the occurrence of long "sticking" intervals near the turning points of the

applied force. As a result of recent follow-up work on this model, analogies have been drawn between the numerical solution of such frictional problems and free boundary problems for partial differential equations. This is one way of avoiding small step sizes near sticking points and after some preliminary theoretical discussion in a current Oxford D.Phil. thesis it is hoped that a CASE student will explore this further.

It was a pleasure to see Mr. Hoad for a third time to discuss the analysis of furnace reactions. Assuming a first order reaction in a furnace whose wall temperature is being uniformly increased, the problem is to infer the activation energy and heat of reaction from a knowledge of the temperature history of the sample. It is interesting to know if there is any analogue of the criticality which can occur in autonomous systems, causing the furnace temperature suddenly to deviate from the imposed wall temperature. This is currently the topic of an M.Sc. dissertation in Oxford.

Dr. Purcell from RAE brought the first optimisation problem to be considered at the Study Group. His optimal traffic scheduling problem provoked such forcefully expressed comments that it was difficult to compare the different suggestions. Two proposals were eventually implemented, one of which, by replacing the separable objective function by piecewise linear approximations, reduced the problem to one of linear programming.

Mr. Booth from BP also returned for a third visit, this time with Dr. Nebrensky to discuss foam break-up in cyclones. This was a difficult modelling problem and the only contribution made at the Study Group was a general discussion of single phase flow in conical cyclones together with a review of possible mechanisms for foam collapse.

Dr. Count from CEGB, Marchwood gave a most interesting description of the problems encountered in transferring wave power from a floating cam through a hydraulic linkage. This again highlighted the importance of frictional oscillations, now in a system with dispersion. The way in which the linkage can excite higher frequencies remains an interesting open question which we hope will be studied at Oxford and Nottingham. Dr. Count's description has also prompted analysis of other cam shapes at Shrivenham.

Mr. Halliday from BAC returned with Dr. Cole to discuss yet another frictional oscillation problem, this time in connection with flutter, so

that the aircraft velocity appeared as the parameter on whose dependence the limit cycle amplitude was sought. This problem was analysed at the Study Group by the describing function technique which gave accurate and economical results even for fairly involved problems; corroborative work was also done on a multiple scale solution for small friction. There has been considerable follow-up work on related problems with other types of nonlinearity.

Mr. Kershaw from Plessey, Templecombe was concerned with a further generalisation of the elastic plate/fluid interaction described by Dr. Butler in 1974. Some numerical work had already been successfully carried out in Oxford on Dr. Butler's problem and related topics of interest to him, but Mr. Kershaw posed the problem of fluid loaded parallel plates acting as a waveguide. It was suggested that some of the information contained in his transform solution could be extracted more easily using a ray method approach and the results of this investigation are appearing in an Oxford M.Sc. dissertation.

Dr. Lewis from B.S.C. Ladgate Lane brought two finite element stress calculations, one involving the bending of a plate on a concrete foundation and the other the compression of a slab confined between parallel walls. Both these problems involved unknown boundaries at which contact was broken or slipping occurred respectively and several iterative schemes were suggested for finding these boundaries. In particular the use of a new variational formulation seemed to promise economies in the solution of the second problem.

Mr. Moore from B.S.C. Grangetown was interested in steady state reacting gas flow in a packed catalyst column. This was a problem similar to Mr. Massey's but one with which much more progress was made, partly because of the availability of the relevant thermal parameters. The dimensionless groups were such as to permit a preliminary boundary layer analysis at the Study Group. This work has since been modified in the light of numerical work carried out at Oxford, the numerical alternating direction implicit scheme itself having benefited from the early analytical work.

Mr. Shipley returned from ICI Billingham with a problem of periodically reversed gas flow in a porous regenerator. He had already obtained a satisfactory solution to the integral equation of the second kind to which his model hyperbolic system could be reduced and all the Study Group provided were asymptotic analyses which could be used to verify Mr. Shipley's results in two different limits.

Mr. De Zoysa from NEL had encountered iteration difficulties with the two point boundary value problem describing the equilibrium of an anchor cable. This provoked lively discussion at the Study Group about the iterative solution of nonlinear equations. Two successful proposals were made; the first made use of an analytical solution to provide a good initial guess when the cable is nearly taut, while the second introduced a relaxation parameter into the iteration to ensure that the physical constraints were met at each stage.

3. Other Contacts

There was a brief correspondence with Pilkingtons on numerical solutions for elastic flexure of large windows and a working session was convened with GEC to consider the finite element solution of Maxwell's equation for linear induction motors. There was considerable discussion with Plessey, Caswell, on the stability of flows occurring in glass fibre extrusion and the relevance of work done at Imperial College. Brief discussions were also held with Plessey on optical tachometer mirror design, with CERL on particle trajections in tidal flows and with Alcan on general problems of heat and mass transfer.

A particularly fruitful contact has been with Formica International concerning the high pressure application of resin to paper by doctor rolls. Over a period of two years the relevant modifications have been successfully carried out to classical lubrication theory to cater for the porosity and compressibility of the paper. Although agreeing in general with the experimental results, the analytical and numerical solutions indicated some surprising parameter dependence.

The Study Group has continued to be deeply involved with unknown boundary problems of various sorts. There was successful collaboration with UKAEA Culham, for whom a programme was written and run to compute the depth of melting in aircraft wings by lightning strikes. This problem was noteworthy for being the only Study Group problem to date for which we have been justified in charging a fee for computer time. On a similar topic, discussions were held with Philips on computer models for radiative melting of frozen food. A more complicated phase change problem on which there has been extensive contact with CERL is that of snow melting in the

presence of a pollutant. A preliminary and rather elaborate model of the combined heat and mass transfer effects has been proposed, although even this may not predict "chanelling" phenomena. It has been agreed that a research assistant, financed by the CEGB for two years, be appointed in the near future to analyse this model and its implications. Collaboration with CEGB, Marchwood on the temperature and velocity distribution in continuous laser welding has resulted in a large part of a current Oxford D.Phil. thesis being devoted to this topic.

Lengthy discussions have been held with ICI Mond on problems of modelling electrolytic membranes, analysing flows in brine evaporators and modelling crystal growth therein, and studying reactive diffusion of water vapour in detergents. The latter two areas are of considerable mutual interest and a CASE award has been allocated to the evaporator problem.

Preliminary contact has been made with Seismograph Services Ltd. on the evaluation of Fourier integrals arising in the analysis of seismic data and a working session was held with Esso in which fuel splashing was identified as a possible area for future collaboration. Finally, general discussions have been held with the Health & Safety Executive on radiative heat transfer and techniques for noise measurement.

4. Discussion

Two important changes have taken place during the past three years. First, a comparison of this report with the previous two indicates a sharp increase in the number of problems submitted to the Study Group. Indeed, the attendance is now so large that accommodation has been transferred from St. Catherine's College to the Department of External Studies. In view of the danger of saturation, it is perhaps a slight relief that the experimental introduction of optimisation, operations research and statistics in the advertisements for 1975/6 failed to produce much response. Secondly, the Study Group has acquired much greater stability since Oxford University took over the post of secretary. However, the administrative and travel expenses still have to be met from external sources such as the Science Research Council, and, as explained below the Study Group activities are changing in such a way that the

scale of these expenses must increase for the Group to retain its effectiveness.

There have recently been three clear signs of the success of the Study Group idea. First has been the growing number of industrial research workers who have reappeared two or even three times with new and interesting problems. Second has been the successful organisation of three workshops, more or less on Study Group lines. One was held in Oxford and one in Cambridge, both CEGB sponsored, and one in Manchester. These workshops have involved larger numbers of people with more concentrated programmes than those at the Study Group weeks, but have successfully emphasised follow-up collaboration. Third has been the greatly increased cooperation of British academic mathematicians from outside Oxford. This has arisen partly from personal contacts and partly from a presentation which was made at the Edinburgh British Theoretical Mechanics Colloquium, when the organisers kindly allotted the Study Groups a prominent position in the programme. It is a great pleasure to acknowledge the help we have received from Brunel, Leeds, Imperial College London, N.P.L., Nottingham, Reading and Shrivenham, without which the above reports would have been far less positive. It is to guarantee the continuation of this cooperation that the appropriate level of funding for expenses for the Study Groups is so important. Support from foreign academics continues at its previous encouraging level.

As in .1974, the continuing backlog of problems remains the principal difficulty facing the Study Groups. While any decrease in the numbers of faculty, inside and outside Oxford, and students working on Study Group problems would cause a dramatic increase in the backlog, there are three reasons for optimism for the future. First, in addition to the continuing student enthusiasm for Study Group problems at the D.Phil. level, from 1978 onwards there will be a new Oxford M.Sc. sub-titled "Mathematical Modelling and Numerical Analysis". It is hoped that this will attract up to 15 students each year, most of whom will be able to write dissertations on aspects of Study Group problems. Second, is the possibility of there being in Oxford several new postdoctoral appointments in areas of activity related to the Study Group. During the next year there will be three such posts, two sponsored by the Science Research Council working on problems involving free and moving

boundaries, and one sponsored by the CEGB on the snow melting problem described above; the existence of all these posts can be traced back to the identification of the importance of moving boundary problems by the Study Group in 1972-4. Third, and in the secretary's view the most encouraging feature of all, has been the way in which the experience of the early years of the Study Group has recently begun to help in drawing analogies between different industrial problems. This not only makes for a more efficient and speedy operation, but has allowed us to pinpoint new areas of research at and beyond the D.Phil. level. In particular the problems of Butler, Booth (ii) and Kershaw have indicated the need for more research into the acoustics of fluid loaded elastic structures; those of Halliday (i) and (ii) and Count have identified the need for greater understanding of frictional oscillations and prompted the analogy with moving boundary problems alluded to above; and those of Massey, Moore and ICI Mond have highlighted the importance of efficient analytical and numerical methods for reacting flows in porous media. It is the mathematical interest inherent in these types of problems which stimulates faculty and students to become involved, and it is upon this involvement that the success of the Study Group depends.

J. R. Ockendon
August, 1977

Appendix 1

Industrial Participants

Alcan International, Banbury.
Babcock & Wilcox, Renfrew.
British Aircraft Corporation, Warton, Lancs.
British Petroleum, Sunbury.
British Steel Corporation, Grangetown, Middlesbrough.
British Steel Corporation, Ladgate Lane, Middlesbrough.
Central Electricity Generating Board, Marchwood, Southampton.
Central Electricity Research Laboratories, Leatherhead.
Esso Research, Abingdon.
Formica International, Maidenhead.
General Electric Company, Hirst Research Centre, Wembley.
Health & Safety Executive, London.
ICI Agricultural Division, Billingham.
ICI Mond Division, Runcorn.
National Coal Board, Ashby.
National Engineering Laboratory, East Kilbride.
C. A. Parsons Ltd., Newcastle.
Philips, Bedford.
Pilkington Bros., Latham.
Plessey, Caswell, Northants.
Plessey, Templecombe, Somerset.
R. A. E. Farnborough, Hants.
Standard Telecommunication Labs. Ltd., Harlow.
Seismograph Service Ltd., Farnborough, Kent.
UKAEA, Culham.
UKAEA, Risley.

Appendix 2

Faculty Participants

Oxford
Dr. D. Acheson
Dr. J. D. Donnelly
Dr. C. Elliott
Prof. L. Fox
Dr. I. Grant
Prof. A. E. Green
Dr. D. Handscomb
Dr. S. McKee
Dr. D. Mayers
Dr. J. D. Murray
Dr. H. Ockendon
Dr. J. Ockendon
Dr. M. Rayner
Dr. A. B. Tayler
Dr. K. Thomas
Dr. A. Tomaras
Prof. L. C. Woods
and about twenty students

U.K. outside Oxford
Mr. E. Albasiny, N.P.L.
Mr. Anand, Middlesex Poly.
Dr. R. Collins, Salford
Dr. A. Crowley, Brunel
Mr. A. Curtis, N.P.L.
Mr. D. Ferriss, N.P.L.
Mr. A. Findlay, Middlesex Poly.
Mr. P. Gill, N.P.L.
Dr. W. Green, Nottingham
Dr. D. Ingham, Leeds.
Mr. G. Miller, N.P.L.
Prof. K. W. Morton, Reading
Mr. P. Nesbitt, Middlesex Poly.
Prof. P. Parks, Shrivenham
Prof. J. R. A. Pearson, Imperial College
Dr. T. Rogers, Nottingham
Dr. F. Smith, Imperial College
Dr. B. Stonebridge, Bristol
Mr. G. Symm, N.P.L.

Visitors from Abroad
Prof. J. Douglas, Chicago
Prof. J. Jerome, Northwestern
Prof. D. Larson, Bloomington, Indiana
Prof. O. Mangasarian, Madison
Prof. L. Segel, Weizmann Inst., Rehovot
Dr. J. Snyman, Pretoria
Prof. H. Stetter, Vienna

Appendix 3

Publications relevant to Study Group Problems

Fox, L. 1976 Numerical Analysis, Computers and Problem-Solving. Interdisc. Sci. Rev. **1**, 2, 167.

Farmer, D. & Ockendon, J. R. 1976 CEGB/Oxford Mathematics Teach-in CERL Report, Leatherhead.

Handscomb, D. C. 1977 Vibrations of a submerged window. J. Inst. Math. Applics. (in press).

Crowley, A. B. & Ockendon, J. R. 1977 A Stefan problem with a non-monotone boundary. J. Inst. Math. Applics. (in press).

Ockendon, H. & Ockendon, J. R. 1977 Variable viscosity flows in heated and cooled channels. J. Fluid Mech. (in press).

Ockendon, J. R. 1977 Numerical and asymptotic solution of moving boundary problems. To appear in Proceedings of Gatlinburg Conference, September 1977.

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OXFORD STUDY GROUPS WITH INDUSTRY

1978-1980

Fourth Progress Report On Applications of Differential Equations

Mathematical Institute, Oxford

1. Introduction

This report describes the activities of the Oxford Study Groups with Industry during their fourth three year period of operation. Support from the Science Research Council ceased at the end of the period but the continuation of the activities for a further period has been assured by donations from BAC, BP, BSC, CEGB, ICI and Plessey. The basic aims of helping research workers in industry and enabling academic applied mathematicians to work on practical problems have remained unchanged, but a further objective of developing a training programme for students wishing to work in industry has been added. Considerable interest in the Study Group activities has been shown by groups in North America, Europe and Australia who are initiating projects with similar aims.

Following the pattern of previous reports the new experience gained over the past three years will be reviewed here after a chronological listing of the problems submitted during that time. Participants from both industry and university, and publications including theses relating to Study Group problems, are listed in Appendices. More detailed information about any of the problems can be obtained from the Study Group Secretary, Dr. J. R. Ockendon, Mathematical Institute, 24-29 St. Giles, Oxford.

2. Problems Discussed at Study Groups

Mr. Bearon (Pilkington) brought the problem of lens polishing on a rouge layer above a viscoelastic pitch "lap". There was considerable discussion of thin layer approximations both for the rouge hydrodynamics and the pitch deformation but even after a follow-up meeting there are still two alternative models for the rouge layer.

Mr. Wolfenden (Courtaulds) was interested in instabilities in synthetic fibre drawing with heat addition and one phenomenological model was proposed at the Study Group. Its analysis was fortunately simple enough to disclose linear instability under certain conditions, but other models may have been more appropriate.

Mr. Gibb (CEGB, Marchwood) wished to know the lifetime and tra-

jectory of an evaporating and radiating fuel droplet. There was only time at the Study Group to produce a preliminary list of possible mechanisms to be modelled, but this revealed several interesting possibilities apparently untouched in the academic literature.

Mr. Wilkinson and Mr. Jackson (C. A. Parsons) returned with the condenser problem which had been considered at the 1975 Study Group. It was soon realised that only a mathematically singular condenser design had been considered at that meeting and the behaviour of the model under more general conditions has now been assessed.

Mr. Watts (BSC, Swinden) brought the long-standing problem of hot rolling of steel. This time discussion centred on the numerical solution of his full "nonlinear fluid" model and its asymptotic solution (i) for small deformations, which led to dual integral equations and (ii) in the "thin layer" limit.

Mr. Burrow (Courtaulds) was interested in "tow spreading" which is the phenomenon which occurs when a bundle of initially parallel, closely packed fibres is immersed in a liquid flowing parallel to them. In equilibrium, the stationary fibres fan out radially but the mechanism for this spreading was not identified at the Study Group. Therefore, the important question of the stability of a spreading bundle could not be answered.

Mr. Challis and Mr. Patrick (Triplex) brought the problem of the deformation under gravity of a hot windscreen supported at points around its perimeter. This provoked a wide-ranging discussion of the relative merits of different types of viscoelastic models but the only concrete calculation to be performed was on an elastic model under ad hoc assumptions about which moments most affect the sag.

Miss Broadhurst (RAE, Farnborough) was concerned with the numerical solution of the unsteady transonic flow equations, linearised with respect to a known steady transonic solution. There was some discussion of the modelling of shocks and of the best numerical methods for inverting the very large matrices resulting from finite-difference discretisations.

Mr. Moore (BSC, Grangetown) was concerned with the flow of a high pressure cool gas jet into a bath of liquid steel, the jet entering through a narrow channel (tweer) in the refractory wall. A very crude "Stefan" model of this complex situation was constructed in which the gas/solid interface near the wall was considered known and the development in time of the liquidus and solidus was computed.

Dr. Hood (BP) was interested in the effect of weak inhomogeneities on the seismic detection of scatterers such as abrupt

density discontinuities. The Study Group discussions centred on the implications of ray theory for this situation and some simple examples of the "direct" problem were calculated in the hope that they might begin a useful catalogue of explicit solutions. Several further discussions have been held with Dr. Hood in Oxford.

Dr. Farmer and Dr. Shephard (UKAEA Culceth) were concerned with the numerical computation of thin flame fronts, but the only aspect of this problem to emerge at the Study Group concerned the jumps in the flow variables across the flame. These should be consistent with the large activation energy limit of the relevant reaction-diffusion equations, and this limit was explicitly demonstrated in some simple cases.

Dr. Berz and her colleagues (Philips, Redhill) brought a model for two-dimensional effects in a p-i-n diode. This model comprised a pair of coupled nonlinear elliptic equations. Both numerical and analytical solutions were discovered during the Study Group and these were of sufficient interest to warrant further consultation during the summer, when two quite different numerical schemes were developed in Oxford and used to predict diode performance.

Dr. White (CEGB Marchwood) was interested in the analysis of a heat flow model for an aquifer with a view to developing a theory of aquifer location. One asymptotic limit considered at the Study Group was when the aquifer was shallow, when a simple relation was discovered between the aquifer depth and the heat flux at ground level.

Mr. P. Moore (BSC) was interested in the problem of electromagnetic stirring of liquid metals in cylinders around which magnetic pole pieces were rotated. In particular he was anxious to identify those angular velocities and field strengths which produced the most effective stirring. The relevant dimensionless parameters were identified at the Study Group and some preliminary orders of magnitude estimates were made; in addition some advice was given about how to avoid instabilities with shooting methods. However the discussion was bedevilled with ignorance about whether turbulent flow was likely to occur and a follow up meeting was held at Bristol where relevant experimental work had been carried out by Dr. Richardson.

Mr. Wolfenden (Courtaulds) returned with the problem of heating and consequent contraction of a hank of man-made fibres whose initial water content is a third of their volume. The stability of the contraction process is of most interest but there was only time at the Study Group to make a tenuous analogy with heat transfer problems

involving phase changes. At a follow up meeting in Coventry, a time-scale discrepancy appeared in this analogy and this has not yet been resolved.

Mr. Chleboun (Rolls -Royce Aero, Filton) was ultimately interested in turbulent chemically reacting flow in a combustion chamber but the discussion at the Study Group centred on the best methods for discretising Laplacian operators near the axis in a cylindrical polar coordinate system. This problem is one of an increasing class of numerical problems which appear at Study Group meetings where part of a larger package is apparently failing and a remedy is sought which not only resolves the problem but also does not adversely affect the rest of the package.

Mr. Nesvabda (Min.Agric.Fish.Food, Aberdeen) was interested in both the modelling and numerical solution for the temperature in freezing fish, chiefly with an eye to estimating the dependence of the thermal parameters on the temperature. Several suggestions were made concerning the "enthalpy" method for integrating the model and minimization techniques for parameter estimation. However, all these suggestions were tentative and Mr. Nesvabda returned with the problem to the next UCINA meeting. It was hoped that some of the ideas would be of interest to Mr. Davis and Dr. Jones (Unilever, Sharnbrook) who are concerned with very similar problems involving frozen meat.

Dr. Ralph and Dr. Duggan (Philips, Redhill) were concerned with obtaining as concise an approximate solution as possible to the generalized telegraph equation which describes the excess free hole density in a semiconductor when trapping occurs. They had already proposed an ingenious convolution representation and all that remained to be done at the Study Group was to confirm (using matched asymptotic expansions) that this representation was indeed asymptotically correct in the two parameter ranges of most interest.

Dr. Furzeland (Shell, Thornton) attended the meeting very briefly to discuss some general aspects of bubble collapse but there was time to do little more than list possible relevant mechanisms. Gas dynamic effects in the bubble, compressibility effects in the liquid, latent heat release and wall effects were all mentioned and it was hoped that Shell would be able to identify some of these effects as being much more important than others.

3. Other Contacts and Activities

A number of visits have been made by Study Group members (usually the Secretary and a graduate student) to industry both to

follow up problems presented at Study Group meetings and to discuss new problem areas; and reciprocal visits have been made to Oxford. New problem areas which have been discussed in detail are flow in a strongly stratified brine evaporator (ICI), pressure surges in pipes containing bubbly liquids (CERL), heat transfer in materials with rapidly varying thermal properties (Rolls-Royce, Derby) and wave motion in layered media (Plessey). A number of smaller problems discussed include pipe corrosion (Esso), criticality of furnace reactors (ICI), a problem in commutator manufacture (GEC) and impurity diffusion in alloys (CEGB, Berkeley).

The formation of UCINA (Universities Consortium in Industrial Numerical Analysis), with its Secretary based in Oxford and its aim to give help with numerical problems in industry, has resulted in many further enquiries from industry being made to Oxford. Many of these enquiries involve consideration of modelling in addition to numerical analysis so that Study Group members have naturally become involved and a further source of problems has been created. These include combustion computer packages (Rolls-Royce, Filton) and reactor safety computer packages (UKAEA, Springfields).

The Study Group has welcomed to Oxford a number of faculty visitors (Appendix 2) interested in participating or observing the Study Group activities, possibly because they were proposing similar developments in their own universities. Some of them have made significant contributions and have given valuable objective advice. Members of the Study Group have given lectures about its activities in eight countries, and four general articles have been solicited (Appendix 3).

4. Educational Objectives

One feature of the Study Group project since its inception has been the enthusiasm with which graduate students have attended the meetings. More than twenty M.Sc. and ten Ph.D. theses have been written on topics related to Study Group problems. Most of these graduates have found that their Study Group experience has helped them to find an interesting job in industry and that industry has been keen to employ them. The Oxford Faculty of Mathematics, recognising the educational value of the Study Group interaction for students, has set up a one year M.Sc. course called Mathematical Modelling and Numerical Analysis, designed for students wishing to make a career in applied research in industry or government establishments. The first course started in 1978 with five students, there were seven in 1979, and nine are due to start in 1980. The

course is supported by the Science Research Council providing one year grants, and in addition to lecture courses and attendance at a Study Group meeting, students write a thesis on a mathematical topic related to an industrial problem. Lectures and supervision are provided by faculty members of the Study Groups. Considerable interest has been shown in this course by a number of universities in North America where there is currently strong pressure to make university courses in Mathematics more applied.

5. Discussion

The significant increase in activity described above has only been possible because of the effort put in by a few members of the Study Groups and help from visitors. Further assistance in Oxford is required if the current level of activity is to be maintained and, since permanent appointments are not feasible at this time, two new postdoctoral research assistant appointments in the area of mathematical modelling are urgently being sought. An interaction with Nottingham University Theoretical Mechanics Department is planned which should provide experienced faculty help on a regular though distant basis, and a one day follow up meeting on problems from Courtaulds will take place in Nottingham in September 1980. A proposal has been made to CEGB, Marchwood for one of their research staff to visit Oxford on a regular basis throughout 1981 which may also help with the shortage of experienced faculty members' time.

The response from British industry has increased slightly over the past three years but there is still considerable scepticism on the part of management about the value of mathematics in industrial research. Thus in times of recession, when the budgets of industrial research departments are cut, it is likely that the longer term, more theoretical research will be pruned. This may however have the result that there will be more demand for free assistance in problems involving mathematics and cheap collaborative projects involving mathematics students. If the Study Groups are to continue to satisfy the present level of demand then new funding will be necessary to support research staff and provide administrative help. If it is not forthcoming in the next two years then a policy of rejecting a significant proportion of the problems presented will have to be reluctantly implemented if the success rate is to maintained.

J. R. Ockendon
A. B. Tayler

September 1980

APPENDIX 1

Industrial participation
(including number of attendances)

Pilkington Bros. Ltd., St. Helens
Courtaulds Ltd., Coventry (3)
CEGB Marchwood, Southampton (2)
BSC Grangetown, Middlesbrough (3)
CA Parsons, Newcastle
Esso, Fawley
ICI, Mond
CEGB, Leatherhead
GEC Hirst Research Centre, Wembley
British Petroleum, Sunbury
BSC, Swinden
UKAEA, Risley
RAE, Farnborough
UKAEA, Culceth
Philips, Redhill (2)
Triplex, Birmingham
Min. Agric. Fish. Food, Aberdeen
Shell, Thornton
Unilever, Sharnbrook, Beds.
Rolls-Royce Aero, Filton
CEGB, Berkeley
Rolls-Royce, Derby

APPENDIX 2

Faculty Participants

Oxford Faculty and Post-Doctorals

Dr. H. Ockendon
Dr. J. Ockendon
Dr. A. B. Tayler
Prof. L. C. Woods
Dr. D. J. Acheson
Prof. A. E. Green
Prof. T. B. Benjamin
Prof. L. Fox
Dr. J. D. Donnelly
Dr. D. F. Mayers
Dr. C. M. Elliott*
Dr. J. Aitchison
Dr. D. Spence
Dr. A. Crowley*
Dr. R. Furzeland*
Dr. R. Turner*
Dr. A. Lacey*
Dr. J. Norbury
Dr. J. Rollett
Dr. M. Cooper
Dr. I. Mason

Visitors

Prof. E. Varley,	Lehigh
Dr. D. Parker	
Dr. T. Rogers	
Dr. W. Green	Nottingham
Dr. S. Hibbert	
Dr. A. England	
Dr. D. Harris	
Prof. E. Cumberbatch*	Purdue
Mr. D. Ferriss	NPL
Dr. P. Curtis	
Prof. P. Parks	Shrivenham
Dr. G. Pagan	
Dr. A. Crowley	
Prof. K. Morton	Reading
Dr. Luffman	Queen Elizabeth College, London
Prof. D. Larson*	Indiana
Dr. A. Fowler	Trinity College,
Prof. J. Miller	Dublin
Dr. M. Mortell	
Dr. K. Thomas	Southampton
Dr. V. Hart*	Queensland
Dr. P. Martin	Manchester
Prof. C. Coleman*	Claremont
Prof. E. Elderkin*	
Prof. J. Mahony*	Perth
Prof. R. Sack	Salford
Dr. D. Bose	Brighton Poly.
Dr. C. Elliott	Imperial
Prof. J. Blackwell*	London, Ontario

* *now left*

* *sabbatical*

APPENDIX 3

Publications relevant to Study Group Problems

Tayler, A. B., Fluid flow between a roller and absorbent compressible paper. Q.J.M.A.M., *31* (1978).

Ockendon, J. R., Numerical and asymptotic solution of moving boundary problems. In Moving Boundary Problems (Eds. Wilson D. G., Solomon A. D. & Boggs P.T.). Academic Press 1978.

Crowley, A. B., Numerical solution of Stefan problems. Int.J. of Heat Mass Transfer, *21* (1978).

Ockendon, J. R. & Crowley, A. B., The numerical solution of an alloy solidification problem. Int.J. of Heat Mass Tranfer, *22* (1979).

Ockendon, H., Channel Flow with temperature dependent viscosity and internal viscous dissipation, J.F.M., *93* (1979).

Crowley, A. B., On the weak solution of moving boundary problems. J.Inst. Math.App., *24* (1979).

Lacey, A., Spatial dependence of supercritical reacting systems. To appear in J.Inst.Math.App. (1980).

Lacey, A., Moving boundary problems in the flow of liquids through porous media. Submitted to Q.J.M.A.M.

Lacey, A., Ockendon, J. R., & Tayler, A. B., 'Waiting time' solutions of a nonlinear diffusion equation. Submitted to S.I.A.M., J.App.Math.

Tayler, A. B., & Nicholas, M., Unsteady slow flows over a cooled flat plate. Submitted to J.Inst.Math.App.

General Articles

Tayler, A. B., Graduate Education using Industrial Research Problems. S.I.A.M. News, *12* No. 3 (1979).

Ockendon, J. R., Differential Equations and Industry. The Mathematical Scientist, *5* (1980).

Tayler, A. B., Mathematical Modelling and Numerical Analysis; Oxford Study Groups with Industry 1968-79. Bull.Inst.Math.App., *16* (1980).

Tayler, A. B., Mathematical Modelling: Art or Science? The Mathematical Intelligencer *2* (1980).

APPENDIX 4

Theses related to Study Group Problems

D.Phil.

1978	Please, C. P.	Some mathematical problems of semiconductor devices.
1978	Rogers, S.	Some aspects of Stefan type problems.
1979	Lacey, A. A.	Some problems in heat and mass transfer.
1980	(Submitted) Bruce, S.	Stratified flows in a brine evaporator.

M.Sc.

1978	Troup, J. E.	Analysis of a fluid-plate interaction problem.
1978	Broadhurst, D. J.	The effect of floating viscoelastic mats on surface gravity waves.
1979	Hall, K. A.,	Asymptotic methods for wave motion.
1979	Nicholas, M.	Numerical solution of singular diffusion equations.
1980	(Submitted) Stewart, G.	Applications of similarity solutions of the percolation equation.
1980	(Submitted) Howison, S.	Free surface flows at low Reynolds numbers.

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OXFORD STUDY GROUPS WITH INDUSTRY

1980 – 1983

**Fifth Progress Report On
Applications of Differential Equations**

Mathematical Institute, Oxford

1. Introduction

This report describes the activities of the Oxford Study Groups with Industry during their fifth three-year period of operation.

Despite some recent changes in operating style, the aims of the Study Groups remain

(i) to help industrial research workers solve problems in physical applied mathematics,

(ii) to involve academic mathematicians in practical problems,

(iii) to provide suitable training for graduate students who wish to work in industry.

To these ends, annual week-long meetings are held which involve about ten industrial and fifty academic participants including graduate students. The industrial participants attend either through established contacts or by responding to a circular sent each December to about two hundred organisations in Britain. New participants are especially welcome and to encourage this the cost of attending is restricted to travel and living expenses. Further details can be obtained from the Secretary, Dr. J. R. Ockendon, Mathematical Institute, 24-29 St. Giles, Oxford OX1 3LB.

Following the pattern of previous years, this report contains brief descriptions of the problems brought to the last three years' meetings. Then follows an account of other contacts and activities which is much longer than in the earliest reports. This expansion of activity has come about through the success of the University Consortium for Industrial Numerical Analysis (UCINA) in soliciting new industrial problems which have an applied mathematical as well as numerical content. The list of academic visitors is also much longer than in previous booklets, and this increase reflects growing interest at home and abroad in university/industry collaboration. Within the past three years, Study Group problems have been discussed in Oxford with visitors from at least twenty-five other universities and this activity has been so diverse that, unlike earlier progress reports, not all the collaboration has been listed here.

Industrial sponsorship throughout the period has largely come from the C.E.G.B., who have also made a large contribution towards the continuity of the important chair of numerical analysis in Oxford, and we also gratefully acknowledge donations from Shell and Plessey.

The S.E.R.C. has continued its support of the Study Groups by providing funds for a full-time postdoctoral assistant Dr. M. Shillor.

2. Problems Discussed at Study Group Meetings

Drs. Furzeland and Woodhead (Shell) were concerned with the depressurisation of liquid filled pipes but only a very crude discussion of the heat and mass transfer in the ensuing two-phase flow was possible at the Study Group. Several order of magnitude estimates for the exit times were given, depending on the driving mechanism, and Dr. Whalley from the engineering department provided references to the relevant two-phase flow literature.

Mr. Tucknott (B.S.C., Port Talbot) was interested in predicting macroscopic temperature and impurity variations in solidifying steel ingots, but the discussion veered towards the classical theory of alloy solidification and its stability. Conjectures were made concerning the usefulness of the classical model in predicting 'mushy regions' and these conjectures have recently been proved in the 1983 literature. A possible new model for dendritic growth has since been devised (ref. [14] of Appendix 3).

Mr. Heyda (B.I.C.C.) brought the problem of upwinding wire from a horizontal coil. This complicated problem originated at a UCINA meeting and the 'Love' model for the wire presents numerical difficulties which are still being considered in Oxford.

Drs. Newlands and Please (C.E.R.L.) brought the fascinating problem of seawater percolation in a gently sloping sand bank. The aim was to predict temperature variations resulting from solar heating but the mechanics of the problem proved so interesting as to dominate the Study Group discussion. An asymptotic theory for small sand bank slope was formulated and subsequently solved, [2].

Dr. Hemp (Cranfield) was attempting to classify electro-magnetic flowmeters whose observed potential difference is proportional to the total flow rate and independent of the detailed velocity distribution. This problem was posed as an overdetermined system of partial differential equations at the 1982 Study Group, and some invariance properties were noticed; however, when Dr. Hemp returned in 1983, the problem was effectively solved using a series approach.

Dr. Farmer (Winfrith) was interested in the famous problem of numerical simulation of the unstable immiscible displacement of one fluid in a porous medium by a second less viscous fluid. The well-posedness of the model was discussed and analogues were drawn with other free boundary problems. At a subsequent Study Group meeting, the use of variational inequalities to predict blow-up and cusp formation was discussed.

Dr. Butler and Mr. Thomas (Plessey, Templecombe) were interested in modelling an array of geophones towed behind a ship. The water pulley effect on the cable was modelled using a simple drag formula and a numerical solution of the resulting hyperbolic partial differential equation was carried out by a D.Phil. student. Care was needed because of the singularity caused by the vanishing tension at the end of the cable.

Drs. Furzeland and Woodhead (Shell) returned with the problem of the break-up of a liquid jet under the action of inertia and surface tension. The importance of a counter current airflow around the jet was stressed and recent experimental evidence from Exeter University was cited.

Drs. Please and Preston and Mr. Edwards (C.E.R.L.) were interested in the well-posedness of seemingly classical boundary value problems for the two-dimensional shallow water equations in finite spatial domains, such as estuaries. After a lengthy literature survey and discussion of energy estimates and characteristics, a catalogue of admissible boundary data has now been produced, [4].

Messrs. Evans and Sage (Sir William Halcrow and Partners) wished to construct a model for landslide generated bores in lakes and reservoirs. This problem arose from a UCINA meeting and the discussion centred on the wave interactions which occurred when the landslide stopped, taking into account the nonuniformity of the lake. A subsequent investigation of the model was carried out by an M.Sc. student.

It was a pleasure to welcome back Dr. Berz (Philips, Redhill) to discuss a moving boundary problem encountered in the transient operation of a diode. A fixed domain formulation was suggested and implemented the Study Group, [5].

Mr. Smith (T.R.A.D.A.) also came via UCINA to discuss the elastic/plastic modelling of timber with special reference to joints and structural elements. The Nottingham participants were able to give him a review of those elements of solid mechanics which could possibly be relevant and a report has now been written on this subject.

Mr. Metcalfe (Courtaulds) came with the problem of evaporation from viscous fluid flowing down the outside of a set of vertical cones. A thin film model was constructed both for a de-aerator (using free surface lubrication theory and a bubble trajectory model) and a de-monomeriser (using a Darcy law for flow in a foam running down the cones). Residence times for bubbles were estimated in each case.

Mr. Moore (B.S.C. Teesside) returned with the very difficult problem of finding numerical solutions to the Navier-Stokes equations at moderate Reynolds numbers in the presence of electro-magnetic body forces. Some numerical advice was given by Dr. Craine of Southampton University and by members of UCINA.

Drs. Furzeland and Woodhead returned with yet another unconventional problem, namely the modelling of partially submerged flexible pipes; this was reminiscent of the anchor cable problem of

1977. The proposed system of ordinary differential equations was found to suffer from ill-conditioning and examples of sensitive dependence on initial data were presented.

Mr. Metcalfe and Dr. Adams (Cathodeon) brought, via UCINA, the intriguing problem of automatic frictional grinding of quartz crystals using rotating drums. This untouched area of modelling involves the study of granular materials, nonlinear oscillations and wear. Some very crude ideas put forward at the Study Group have since been refined to give more insight into the mechanics of the operation. Collaboration with D.A.M.T.P. Cambridge is envisaged.

Dr. Briggs (Courtaulds) requested an analysis of a complicated ordinary differential equation model for a polymeriser, an extension of the previous year's problem. The stability of the dynamical system was discussed, thereby suggesting a possible control system, and further work in this area may be carried out at the University of Warwick.

Dr. Nicol (M.E.L.) returned to Oxford with the topical problem of hydrofracture of hot dry rock. Although this problem was subsequently found to have a lengthy literature, a model for the crack opening was proposed which may avoid the bugbear of large negative pressures near the crack tip. Further collaboration with groups in New Zealand and at Imperial College has been arranged.

Dr. Butler and Mr. Knight (Plessey, Templecombe) were concerned with the novel problem of wave motion in a membrane stiffened by a periodic array of parallel strings. Despite its apparent simplicity, the Fourier analysis of this problem poses severe difficulties and a report is still being written to emphasise the strange behaviour between the pass and stop bands of the frequency spectrum.

Mr. Jordan (International Paint) came with the challenging problem of modelling electrolytic painting. Fortunately, some preliminary work had been done on this problem which enabled it to be formulated as a variational inequality similar to the C.E.R.L. sandbank model. This work has now been implemented, [6], and further work on fully painted specimens is continuing at Oxford.

Mr. Barber (B.S.C. Teesside) brought another deceptively simple problem involving the bulging of a slab of cast steel being guided by a set of rollers. The hydrostatic pressure of the still molten steel is responsible for the bulging but even the simplest model for the deformation of the solid crust leads to a high order ordinary differential equation whose well posedness is still being studied by an M.Sc. student.

Dr. Musker (A.M.T.E.) visited the Study Group briefly to discuss hydrodynamic forces on obstacles submerged beneath a train of surface gravity waves. A D.Phil. student is considering this problem using matched asymptotic expansions and singularity distributions.

3. Other Contacts

As suggested in the last report, the U.K.A.E.A., Springfields reactor safety study was taken over by UCINA and a two-year post doctoral position was thereby created. Reciprocally, two modelling problems from C.E.G.B. emerged from early UCINA meetings. One from Berkeley concerned the use of squeeze films in mountings for structural members, but surface roughness effects proved difficult to model. The other, from Bedminster Down, Bristol set off a lengthy collaboration on a nonlinear oscillator model for gearbox vibrations.

Two problems which were closely related to each other came from Plessey Marine and C. A. Parsons and involved the summation of slowly convergent series; in each case an improved numerical algorithm was suggested.

A general discussion was held with British Gas in London and contact was then made with the Killingworth Research Station on leak detection in underground pipes. A possible acoustic technique was discussed with Prof. Leppington of Imperial College. A similar venture with B.S.C. at Port Talbot centred on the problem of heat and mass transfer in the flux and powder which is added to the surface of solidifying steel ingots, but this is an area in which several other universities are already involved.

A most interesting analogy arose when a model for electrical painting was proposed for International Paint (Courtaulds) which was almost identical to the C.E.R.L. sandbank problem mentioned earlier. This enabled a numerical solution to be implemented quite quickly and further rapid developments may be possible with Unilever (Sharnbrook) on mechanisms controlling the freezing of food. Here we hope to draw on analogues with solidification in metals and other crystals, an area in which we have considerable experience. The Unilever contact was made via UCINA, who also initiated a discussion of combustion in a granular material degasifier for British Gas. Here a model is required both for the combustion and the granular flow and it is interesting that one of the first ventures of the Duke University group in North Carolina is to organise a meeting on granular materials. (The Study Groups did just this in 1968!) Another UCINA contact was with Plessey (Caswell) on the thermal oxidation of silicon. UCINA are already carrying out a numerical model of a simplified model and a CASE student is about to consider a more complicated description which couples the heat and mass transfer with the stress field. Other CASE projects have been on fluid dynamics and crystallisation in a brine evaporator and on mass transfer in miniaturised counter current flow devices, both for I.C.I. and both completed; a model for coagulation in hanks of man-made fibres is currently under investigation for Courtaulds and it is hoped shortly to commence a fourth CASE project on the mathematical and numerical analysis of a plunging ship bow for A.M.T.E.

(Haslar). As mentioned earlier, collaboration is already taking place with A.M.T.E. on lift and drag forces on submerged bodies.

The range of topics which has been discussed with Marchwood Engineering Laboratories (C.E.G.B.) is too wide to be described here but one problem involving flow in aquifers has involved considerable numerical work in Oxford. A further problem concerning a new design of heat exchanger has also led to a six month research contract.

4. Graduate Training

The M.Sc. course, whose inception was described in the last booklet, has attracted about eight students during each of the past three years, although rather fewer than half of these have subsequently taken up positions in industry. At the moment the advertisement for the course is as follows:

M.Sc. in MATHEMATICAL MODELLING and NUMERICAL ANALYSIS

Since 1968 there has been at Oxford a programme of collaboration with industry on research problems involving differential equations and numerical analysis. This interaction with industry, coupled with dramatic increases in industrial computing power, has demonstrated that there are clear needs for

graduates with mathematical modelling skills,

graduates with numerical skills.

This one year course organised by the Faculty of Mathematics has been designed to meet these needs by training able graduates in mathematics and related disciplines in the application of mathematics to a wide range of scientific and technological problems. Industry strongly supports the course and successful graduates have excellent career opportunities.

The course consists of examined lecture courses and a dissertation, normally based on an industrial problem. Students opt to weight their studies towards modelling or numerical analysis. Typical dissertation titles are

Torsional vibration in gear boxes,

Some applications of the finite element method in solving the Stefan problem,

Flow of granular material in a hopper,

Numerical analysis of open channel flows.

Topics in mathematical biology may also be offered.

Acting on advice from industry and possibly with industrial financial support, it is hoped to improve both the quality of the training and to increase the number of students in the near future. In particular it is intended that more emphasis should be put on the dissertation and the students' ability to communicate results. In addition to

those in the advertisement, topics for modelling dissertations have included the spread of liquid layers under gravity, thermal explosions, bubble motion near boundaries, shallow water flows, flows in regenerative heat exchangers and the deformation of partially solidified steel slabs.

Relevant D.Phil. theses are

1981	Bruce, S.	Stratified Flows in a Brine Evaporator.
1981	Nicholas, M.	Some Numerical and Theoretical Aspects of a Problem arising from Industry
1983	Howison, S.	Free and Moving Boundary Problems in Hydrodynamics
1983	(Submitted) Fitt, V.	Counter-current Mass Transfer

5. Discussion

The continuing increase in activity described in this report has resulted from two main factors; (i) the existence of UCINA with its inevitable spin-off of applied mathematics problems and (ii) the interest shown in the Study Groups by academics outside Oxford who attend the meetings in sufficient numbers and with sufficient enthusiasm that almost any problem posed by an industrial visitor is guaranteed to receive a thorough analysis. This consequent effective increase in manpower has occurred spontaneously and has given the Oxford effort greater stability. Indeed, the new spirit of inter-university collaboration is typefied by a recent one-day problem solving session with D.A.M.T.P., Cambridge at which Study Group problems created considerable interest.

The coexistence of the Study Groups and UCINA, both based in Oxford, mean that they are regarded from outside almost as a single entity and this report describes so many intertwined problems that in practice there is a close affinity between the two groups. It is clearly desirable that the UCINA and Study Groups administrative and secretarial functions should not duplicate each other, but there will be a continuing need for both a modelling and a numerical academic secretary with post-doctoral support in each area. With the external academic support which is now forthcoming, the immediate need for more manpower in Oxford is not so great as it has been over the past ten years, but several of the most active participants at the Study Groups and UCINA do not have tenured positions.

There has been little change in the organisation of the Study Group meetings except for a new programme of group meetings with Marchwood Engineering Laboratories (C.E.G.B.), which have been held every few months for the past two years. This collaboration,

which has successfully involved several different industrial research workers, was devised instead of a secondment scheme which was thought difficult to administer.

On the purely academic side, the most gratifying trend has been the continuing world-wide interest in free and moving boundary problems and the occurrence of regular international meetings on the subject, the very first of which was organised by the Study Group in Oxford in 1974. This area of problems continues to comprise at least a quarter of all those submitted and this fraction is likely to increase, with new applications in crystal growth and metallurgy. Indeed another academic benefit of the Study Group has been the enhancement of interdepartmental collaboration in Oxford, especially with engineering and metallurgy.

On the financial side, within the UK we have been fortunate to attract several donations during the past three years and UCINA has been able to find industrial and government support for at least four more years activity. In the present climate, we hope and have some reason to believe that the quality and quantity of the ideas generated at UCINA and the Study Group meetings will be sufficient to generate sponsorship at a level to cover administrative and support costs.

Internationally there remains the need for collaboration between the different industry-orientated mathematics groups, and an S.E.R.C. grant has been awarded for travel between Duke University (North Carolina), Rensselaer Polytechnic Institute (New York) and Oxford. Such collaboration should certainly stimulate and develop the mathematical ideas emanating from industrial problems.

J. R. Ockendon

October 1983

APPENDIX 1

Industrial Participation
(including number of attendances)

Shell, Thorton (3)
B.S.C., Port Talbot
B.S.C., Teesside (2)
British Insulated Callender's Cables
Central Electricity Research Laboratories (2)
C.E.G.B. Marchwood
C.E.G.B., Berkeley
C.E.G.B., Bedminster Down
Fluid Engineering Unit, Cranfield
Plessey Marine (2)
Plessey, Caswell
A.E.R.E., Winfrith
Sir William Halcrow & Partners
Philips, Redhill
Timber Research & Development Association
Courtaulds (2)
Cathodean Crystals
Admiralty Marine Technology Establishment
U.K.A.E.A., Springfields
International Paint
C. A. Parsons
British Gas, Killingworth
Unilever, Sharnbrook
I.C.I.

APPENDIX 2

Faculty Participants

Oxford	*Visitors*	
Dr. H. Ockendon	Dr. A. K. Head*	CSIRO, Melbourne
Dr. J. R. Ockendon	Dr. L. Wollenberg	Queen Elizabeth College, London
Dr. A. B. Tayler		
Dr. D. J. Acheson	Dr. D. Bose	Brighton Poly.
Prof. L. Fox	Dr. R. Grundy	St. Andrews
Prof. T. B. Benjamin	Dr. D. Riley	Bristol
Dr. J. M. Aitchison	Dr. D. Evans	
Dr. D. A. Spence*	Dr. F. Hodnett	NIME, Limerick
Prof. L. C. Woods	Prof. D. S. Jones*	Dundee
Dr. J. Norbury	Prof. D. Schaeffer	Duke University, North Carolina
Dr. D. C. Handscomb	Prof. D. Kraines	
Dr. D. F. Mayers	Dr. T. Davies	Chemical Eng., Exeter University
Dr. S. McKee		
Dr. J. D. P. Donnelly	Prof. R. Rivlin	Lehigh
Prof. A. E. Green	Prof. E. Varley	
Dr. M. Shillor	Prof. A. Bermudez	Santiago, Spain
Dr. M. Cooper	Dr. A. Crowley	RMCS, Shrivenham
Dr. P. Whalley	Prof. P. Parks	
	Prof. G. Bluman	UBC
	Prof. J. Blake	Woolongong
	Dr. D. Larson	Pacific Sierra Res. Corp.
	Dr. D. Hurley	Perth, Australia
	Dr. G. Keady*	
	Mr. M. Hood*	
	Prof. J. Mahony*	
	Dr. K. Haliste	Umea, Sweden
	Mr. D. Ferriss	NPL
	Dr. C. Series	Warwick
	Dr. R. Craine	Southampton
	Prof. A. Kapila	RPI
	Prof. W. Everitt	Birmingham
	Dr. D. Parker	Nottingham
	Dr. T. Rogers	
	Dr. W. Green	

* *now left*

* *sabbatical*

APPENDIX 3

Publications relevant to Study Group Problems

1. Lacey, A. A., Ockendon, J. R., Tayler, A. B. Waiting-Time Solutions of a non linear diffusion equation. S.I.A.M. J.App.Maths. *42,* 1252, (1982).
2. Aitchison, J. M., Elliott, C. M., Ockendon, J. R. Percolation in gently sloping beaches. I.M.A. J. App.Maths., 1983 (to appear).
3. Cameron, R. F., McKee, S. Direct Numerical Solution of a Volterra integral equation arising out of viscoelastic stress in materials. Comp.Maths. in App. Mech.Eng., *29,* 219 (1981).
4. Edwards, N. A., Please, C. P., Preston, R. Some observations on boundary conditions for shallow-water equations in two space dimensions. I.M.A. J.App.Maths. *30,* 161 (1983).
5. Berz, F., Crowley, A. B., Pritchard, J. Modelling PIN diode switch-off with the enthalpy method. Submitted to J.App.Phys.
6. Aitchison, J. M., Lacey, A. A., Shillor, M. A model for an electropaint process. Submitted to I.M.A. J.App.Maths (1983).
7. Berz, F., Aitchison, J. M. Effect of surface recombination in PIN diodes. Solid-State Electronics. *24,* 795 (1981).
8. Blackwell, J. H., Ockendon, J. R. Exact solution of a Stefan problem relevant to continuous casting. Int.J. Heat Mass Transf. *25,* 1059 (1982).
9. Ockendon, J. R. Survey of Oxford/Industry Problems in 'Computational Techniques for Ordinary Differential Equations'. ed. I. Gladwell, D. Sayers, Academic Press 1980.
10. Elliott, C. M., Ockendon, J. R. Weak and variational methods for free and moving boundary problems. Pitman Research notes in Mathematics, *59* (1982).
11. Lacey, A. A. Moving Boundary problems in the flow of liquid through porous media. J.Aust.Math.Soc. *B24,* 171 (1982).
12. Tayler, A. B. Differential Equations and the real world, Bull.Aust.Math.Soc. *26,* 421, (1982).
13. Ockendon, J. R. Rapporteur in 'Free boundary problems: Theory and application', ed. A. Fasano, M. Primicerio. Pitman Research notes in Mathematics, *78, 79* (1983).
14. Lacey, A. A., Ockendon, J. R., Tayler, A. B. Modelling Mushy Regions, in 'Analysis and Optimisation of Systems', ed. A. Bensoussan, J. L. Lions, Springer Lecture notes in Control and Information Sciences, *44,* (1982).
15. Lacey, A. A., Shillor, M. The existence and stability of regions with superheating in the classical two-phase one-dimensional Stefan problem with heat sources. I.M.A. J.App.Math. *30,* 215 (1983).
16. Lacey, A. A., Tayler, A. B. Modelling Mushy regions. I.M.A. J.App.Math. (1983) to appear.

MATHEMATICAL STUDY GROUPS WITH INDUSTRY 1984 – 1986

SIXTH PROGRESS REPORT ON APPLICATIONS OF DIFFERENT EQUATIONS

Mathematical Institute, Oxford.

1. Introduction

This report describes the activities of the Mathematical Study Groups with Industry during their sixth three-year period of operation. The layout of the report is similar to that of its predecessors, and the Study Group meetings continue to be held in Oxford, but the title of the report has been changed to reflect the increased national and international participation at the meetings. The international scene is discussed further in Section 4; the change in emphasis of the U.K. participation is described in the following extract from the notice for the 1986 meeting:

> 'Applications are invited from research workers in industry and other research establishments who would like to propose scientific problems for discussion at the meeting. Problems may be proposed which involve in some way the formulation, theory or numerical solution of differential equations; the academic participants at the meeting will comprise mathematics faculty members and research students from a number of universities and from UCINA (University Consortium for Industrial Numerical Analysis). In addition to the problem presentation session on the Monday, lectures and seminars will be given on relevant mathematical and computing methods, but as much time as possible will be made available for informal discussions.
>
> The meeting will take place in the Mathematical Institute, Oxford. The format will be based on the eighteen years' experience of the Oxford Study Groups with Industry, further details of which can be obtained from the Secretary, Dr. J.R. Ockendon. This year the available expertise has been increased by the creation of an organising committee with representatives from Cambridge, Heriot-Watt and Nottingham universities.'

The organising committee consists of Dr. W.A. Green, Nottingham, Dr. E.J. Hinch, Cambridge and Dr. A.A. Lacey, Heriot-Watt, in addition to the Secretary, and it was created because the mathematical resources needed to cope with as many as ten problems presented at each Study Group meeting could no longer be provided locally in Oxford.

Following the usual pattern, this report contains brief descriptions of the problems brought to the last three years' meetings, and then an account of other contacts and activities. Collaboration with the University Consortium for Industrial Numerical Analysis remains strong although the departure of the coordinator, Professor S. McKee, to Strathclyde has meant a recent temporary decrease in the UCINA generated demand for modelling and general applied mathematical expertise.

We would again like to acknowledge gratefully industrial sponsorship from CEGB, ICI, Shell and Plessey and also continued SERC support in the form of a valuable post-doctoral position which has now been held by Drs. M. Shillor, S. Howison and J. Dewynne consecutively. We are also grateful for another project-orientated post-doctoral position (Dr. Wilmott) which has been provided by the Admiralty Research Establishment, Haslar. CASE awards have been held by Drs. V. Fitt (ICI), I. Lund (Courtaulds), J. King (Plessey) and Mr. A. Young (Aspinwall & Co.). Some aspects of international sponsorship are mentioned in Section 4.

2. Problems Discussed at Study Group Meetings

Following a meeting early in 1984, Dr. Jones (Unilever, Bedford) became interested in a model for the freezing of iced lollipops. The 'columnar' morphology is similar to that encountered in ingot solidification, and a model was formulated and solved by Drs. Flood and Hunt in the Oxford Metallurgy Department. The detailed analysis near the tip region was posed as an unsolved problem at the Maubisson conference (2).

Mr Cavanagh (Davy-McKee) was concerned with the hydrodynamic and heat transfer properties of a tilting 'hydropad' device for applying a controlled coolant flow to a hot roller. The lubrication theory was similar to that for the 1977 lens grinding problem and the counter current heat transfer aspect has been described in (10, 25).

Drs. Furzeland and Woodhead (Shell) returned with the problem of modelling the motion of soil and vapour following a sudden underground pipeline rupture. There was a brief discussion of hydraulic fracture and a more mathematical investigation of seepage from a pinhole but the data was insufficient for an unambiguous model to be posed.

Dr. Fletcher (AEE, Winfrith) was interested in the interaction of a liquid-metal jet with a bath of water and in particular the stability of a configuration in which there was a break up into a coarse mixture of liquid-metal particles, each surrounded by a protective layer of steam. This problem did not seem amenable to mathematical analysis; instead some experiments were carried out with jets of water directed into paraffin baths and reference was made to research by Professor Bankoff in Chicago.

Dr. Thornton (Courtaulds) presented the very interesting model of airjet spinning of manmade fibres, in which the vorticity entrained by the fibres when they enter the coagulant bath can cause them to tangle. The tension in the fibres resists this instability and some numerical solutions for the balance between the two mechanisms were found. A preliminary analysis of the heat transfer in the airjet above the bath was also carried out using more conventional boundary layer theory. The hydrodynamical aspects have since been the subject of an M.Sc. dissertation in Oxford.

Mr. Metcalfe (Cathodeon) returned with further discussion of his crystal grinding problem, this time armed with more experimental evidence of the 'sloshing' motions caused by varying the drum speed. These phenomena were explained in terms of stick-slip oscillations with different coefficients of static and dynamic friction and a sufficiently detailed description of the oscillation was given for an estimate of the wear to be made. Further research in this topic has been carried out at Nottingham.

Drs. Cook, Dean and Hobson (CEGB, Leatherhead and Marchwood) brought two problems concerning annular gas flow associated with fuel-rod oscillations, one with axial flow and the other with a closed end to the annulus. A thin layer analysis suggested that the possibility of whirling modes depended crucially on the boundary conditions at the end of the annulus. The two extreme limits of lubrication theory and plug flow yielded sets of coupled ordinary differential equations, but the analysis was not carried through to predict whirling except for linearised irrotational flow.

Dr. Cullen (Met. Office) brought the basic problem of weather front modelling. This engendered some lively discussion and some special similarity solutions of the conservation laws were found which explicitly describe 'frontogenesis'. A numerical approach using convexity ideas was subsequently explored in Oxford.

Dr. Butler (Plessey, Templecombe) returned briefly to discuss wave propagation in nearly periodic structures and this led to a subsequent meeting with analysts from Birmingham, Oxford and Topexpress, Cambridge, in which the spectral properties of such structures were discussed. In particular the effect of dimensionality on the concept of pass/stop bands was emphasised.

Dr. Smith (Electricity Council Research Centre) was interested in current leakage from a high tension cable to earth. A novel third order partial differential equation was proposed for the potential and some interesting solutions were obtained numerically and using complex variables. However, the modelling of the 'corona' of ionised gas near the cable was insufficiently clear for unambiguous boundary conditions to be posed.

Dr. Tidman (ICI, Stevenston) brought the problem of modelling underground explosions near a cliff face in iron-ore mining. This problem first appeared at the 1984 CSIRO meeting in Melbourne and two papers (17, 18) have since been written on the hyperbolic system which is a possible model for the rock motion. The diffusion aspects are the subject of a D.Phil. thesis in Oxford.

Dr. Shipley (ICI, Billingham) returned with a model of the 'pressure-swing' absorption process in which a contaminated gas is purified by absorption during flow through a porous matrix. Some explicit solutions were written down for the two-component case but the problem of what were well posed boundary conditions and the implications for a numerical scheme was not resolved at the meeting. Subsequent discussions have been held with members of UCINA.

Mr. Moore (BSC, Teesside) continued with his series of computational fluid dynamics problems by proposing a stream function/vorticity method for two-dimensional turbulent flow, and some numerical suggestions were made.

Dr. Kumar (BSC, Teesside) was interested in the nonlinear Ergun model for gas flow in the porous matrix of a blast furnace. A mathematical shortcoming of his model was pointed out and a Galerkin approach for a variational formulation was suggested in a special case.

Dr. Butler (Plessey) returned with another acoustic scattering problem; this time the scatterer was a finite elastic plate. Low and high frequency and low fluid loading limits were considered but the practical parameters were such that only a numerical solution was feasible. This has since been carried out by Dr. Handscomb at Oxford and further work has been done by Professor Clements at Dalhousie.

Mr. Senior and Mr. Wolfenden (Courtaulds) brought the novel problem of determining the colour of a fabric dyed with a fluorescent dye. A well established empirical radiative transfer model exists which predicts colours in the absence of fluorescence, and an extension of this theory was proposed. In the general case this leads to an inverse problem for a system of ordinary differential equations.

Dr. Tarabad (Perkins Engines) wished to describe the behaviour of an elastic bearing, which is a classical problem in the absence of cavitation. A 'complementarity' model exists for cavitation in the absence of elasticity, and thus a non-local complementarity model was proposed, but its mathematical analysis remains an open question.

Dr. Woodhead (Shell) returned with the complicated problem of heat and mass transfer when a drop of cold liquid came into contact with hot liquid with possible vapourisation of the drop. Many different physical balances were considered in an effort to identify the most dangerous parameter regime.

Dr. P. Hagan (Exxon, New Jersey) was interested, *inter alia,* with the high temperature corrosion of binary alloys but fortunately he had effectively formulated and solved this problem before the Study Group meeting.

Dr. Cassie (NCB, Bretby) wished to model rock deformation and fracture in the vicinity of the face in a deep coal mine. A simple discrete block model was suggested and reference was made to a paper by Dr. England on a possible continuum model in which a crack was opened by a point load. Further elastic/plastic modelling and numerical analysis has been carried out at Oxford by Dr. Reed on a post-doctoral contract (21).

Dr. Jones (Unilever) returned with a fascinating diffusion problem in which a chemical reaction at the boundary produced a novel nonlinear non-local boundary condition. An asymptotic solution was found to describe the evolution, which occurs in two quite distinct stages, and some numerical work has since formed part of an M.Sc. thesis in Oxford.

Dr. Andrews (CEGB, Marchwood) brought the complex problem of welding by metal transfer from a wire in an arc which provides both heat and momentum transfer to the workpiece. A model was proposed which identified several important dimensionless parameters and led to a novel free boundary problem for an elliptic equation, whose mathematical well-posedness has yet to be established. A CASE award is now available in Oxford for further work on this problem.

Mr. Chalmers (MOD, Bath) wished to predict the forces on the hull of a ship slamming into a heavy sea. Some comments were made about the response in a 'beam' model when a moving point force is applied and also about the relevance of the classical 'wedge-entry' theory. The latter enabled some orders of magnitude to be estimated, but a literature search is needed before further work should be done.

Dr. Cifuentes (Thorn EMI) was intersted in improving his model for emission from high pressure sodium lamps. Considerable discussion of the physical mechanisms resulted in a proposal for a simplified free boundary model in which multiple steady states were possible, as well as finite time blow-up. However, Thorn are still concerned that a more elaborate non-equilibrium model is needed near the electrodes.

Mr. Denison (BSC, Teesside) wanted to predict the temperature variations at the base of a ladle containing liquid steel which had time to stratify by cooling from the side walls. A model was proposed in which thermal boundary layers flowed down the walls and the interior isotherms were horizontal. The interior temperature then satisfies a novel hyperbolic equation with non-local boundary conditions.

Dr. Jones (Unilever) returned with some very general questions about modelling of multiphase systems. All that could be provided at the Study Group was a review of possible analytic and numerical approaches to such problems, in increasing order of complexity.

Dr. Tidman *et al.* (ICI, Stevenston) returned to try to improve the earlier rock blasting model, which was predicting excessive gas velocities. One possibility is a more realistic hydrofracture model at the crack tips, using results from the geophysics literature; this results in a locally parabolic behaviour.

Dr. Turner (CERL) brought the notorious problem of wave propagation in two-phase flow. There was considerable discussion of the 'equal pressure in each phase' assumption, with the concommitant non-hyperbolocity of the system of differential equations. A model was proposed in which the bulk and interfacial pressures were distinct and which, using symbolic manipulation, could be proved to be hyperbolic.

Mr. Moore and Dr. Kumar (BSC, Teesside) returned with another aspect of blast furnace modelling, namely the gas flow in the tuyeres. A crude heat transfer model had been proposed at an earlier Study Group but now a more realistic description of the flow was required. An order of magnitude argument suggested that the gas flow in the raceway was almost irrotational and the coupling with the surrounding porous medium such that a constant pressure condition should be applied.

Mr. Wolfenden (Courtaulds) returned with a new version of the air gap spinning problem referred to earlier. Now the principal concern was the mechanics of the flow near the orifice at the base of the bath. Previous work in Oxford had modelled the fibres as a continuum of parallel cylinders and a very simple extension of this theory was made to include small amplitude transverse variation in the fibres. Further work on this model has been carried out in Birmingham and Cambridge.

Dr. Bryan Davies (Beechams) brought one of the few biological problems to be addressed at the Study Group. This concerned the diffusion of drugs through the skin, modelled as a layered medium with rapid mass transfer possible through pores. A coupled system of diffusion equations were proposed and solved numerically; a follow up meeting has been held to describe more accurate numerical results which were subsequently obtained in Oxford.

3. Other Contacts

There has been regular collaboration with Courtaulds on a variety of problems associated with wet and dry spinning and this has resulted in theoretical and experimental advances as described in (T2). Similar collaboration has occurred with ARE, Haslar, where the principal aim has been to provide explicit asymptotic solution for flows around moving submerged bodies for comparison with numerical results (19).

The past few years have seen the creation of several new systems analysis and mathematical consultancy firms, some of whose objectives may overlap with those of the Study Group. Indeed, representatives from P.A. Technology (Royston) and Smith Associates (Guilford) have attended several Study Group meetings as observers. So far this seems to have been of mutual benefit; in particular, Smith Associates have been able to recruit students working on Study Group problems and have fostered applied mathematics in Oxford through sponsorship of a lecture and graduate scholarship.

Study Group members have maintained an interest in continuous casting problems, with visits to BSC at Stockbridge and Port Talbot, and several open mathematical questions remain in this area.

Regular contact has also been maintained with Unilever (Bedford), Shell (Thornton), CERL, CEGB and Plessey (Caswell). At all these places, active applied mathematics groups provide a regular source of problems and employment for graduates. In particular, various aspects of fluidised bed combustion have been discussed in several meetings with Dr. Gibb at Marchwood. Also some of the many problems suggested by Plessey in the area of semiconductor fabrication are described in (23, 24).

The flow of problems from industry throughout the year has increased to such an extent that regular weekly meetings are now held in Oxford to initiate or report on progress with various industrial collaborators. A particularly fascinating problem was posed by STC concerning the modelling and analysis of thermistors; the free boundary limit when the ratio of electric conductivities becomes large is still an open mathematical problem. Another STC problem where future collaboration is likely concerns the underwater unwinding of a coil of wire; this seems to be an extension of a problem posed to UCINA by BICC some years ago. Preliminary contact was made with Elkem (Norway) to discuss modelling the extrusion of graphite impregnated rods, and also with INCO who were interested in radiative transfer around oxide coated metals.

A fruitful collaboration has been started with Aspinwall & Co. (Shrewsbury) concerning various aspects of modelling of landfills; numerous mathematical models suggest themselves but very few theoretical analyses have yet been carried out.

One academic conference has been held in Oxford which was closely related to Study Group activities. This was the Crystal Growth meeting organised by IMA, (P1), whose ancestry can be traced back to the first international free boundary conference in Oxford in 1974. The 1984 meeting was attended by almost as many industrial as academic participants.

4. Discussion

Apart from the continuing steady growth of the meetings in terms of problems (now an avarage of 10) and participants (over 100, usually 15 from abroad), the most striking development has been the appearance of other Study-Group style meetings in Australia (organised and partly sponsored by CSIRO); RPI, Troy, New York (with support from Sloan Foundation); and Dalhousie, Nova Scotia. The organisers of these meetings had all attended earlier Oxford meetings and, with many overlapping problems emerging, there is scope for a transfer of expertise.

The theme 'mathematics in industry' is increasingly fashionable worldwide. Dr. N. Barton, CSIRO Linfield, New South Wales, has recently written a comparative account of collaboration in Europe and Australia; the first international conference on the topic is being held at Claremont, California in January 1987; and a European Consortium for Mathematics in Industry (ECMI) has been set up and has a meeting at Oberwolfach in February 1987. ECMI's objectives are to orchestrate collaboration within the EEC on mathematics for industry, and universities in other non EEC countries such as Austria, Finland and Norway are eager to be associated. The national bodies SIAM, SMAI, GAMM and IMA are jointly organising an industrial and applied mathematics meeting in Paris in June 1987 and the hope is that this will be the first of a 4 year international cycle; the presence of the word industry in the title of such a large meeting is a sign of the times. There are also requests for help and advice from the third world—India, Nigeria, South America, Thailand—and a Study Group type workshop is being planned in China for 1987/8.

All these developments are both stimulating and gratifying. The only drawback is the extra demand made on the Oxford team who could easily become travelling salesmen for mathematics in industry, calling in on Oxford for a few weeks a year to pick up more problems. At least the younger faculty seem to have limitless energy in this direction, and it is encouraging to note the demand for their services.

One final benefit from the current enhanced image of industrial collaboration is the possibility of new sources of funding of work of theoretical importance. The Study Group background is attractive to sponsors of applied mathematical research and the U.S. Army, U.S. Navy and the EEC have been able to support us on contracts of interest both to them and industry.

Finally, it is appropriate to mention the publication of two books directly related to Study Group activities. The first, (B2), is a graduate text distilled from problems presented between 1968 and 1982 and the second, (B1), is a compendium of problems, many of which were presented at Study Group or UCINA meetings. The whole problem of the publication of 'mathematics-in-industry' work is the subject of much debate at the moment; specific new journals are being suggested, some existing journals such as IMA J. App. Math. are actively soliciting industrial contributions, and SIAM has several ideas under consideration in connection with the Claremont meeting mentioned above.

APPENDIX 1

Industrial Participants (with number of attendances)

CEGB Leatherhead	(3)
CEGB Marchwood	(3)
ICI Stevenston	(2)
ICI Billingham	
Shell Thornton	(3)
Courtaulds	(3)
Unilever	(3)
BSC Teesside	(2)
Plessey Marine	(2)
Thorn EMI	
Beechams	
MOD Bath	
TRADA	
Met. Office	
Cathodeon	
Davy McKee	
AEE Winfrith	
Perkins Engines	
Electricity Council	
NCB Bretby	
Smith Associates	
PA Technology	

APPENDIX 2

Visiting Academic Participants

Professor E. Cumberbatch Professor H. Williams	Claremont, California
Professor E. Varley	Lehigh, Pennsylvania
Professor J. Chadam	Bloomington, Indiana
Professor W. Boyce Professor R. O'Malley Professor D. Drew Professor A. Kapila	RPI, New York
Professor M. Miksis Professor B. Pitman	Duke, North Carolina
Professor J. Clements	Dalhousie, Nova Scotia
Professor R. Westbrook	Calgary
Professor H. Rasmussen	London, Ontario
Dr. A. Head	CSIRO, Clayton, Melbourne
Professor J. Blake	Wollongong
Dr. N. Barton	CSIRO, Linfield, NSW
Professor G. de V. Davies	New South Wales
Dr. D. Larson	Pacific Sierra Res. Corp.
Dr. K. Haliste	Umea
Dr. K. Hoffman	Kaiserslautern
Dr. A. Bossavit	Elect. de France
Dr. C. Holland	ONR
Mr. W. Dorfler	Bonn
Miss R. Casselli	Bari

Dr. J. Molenaar	Amsterdam
Dr. R. Semenzato	Padova
Professor S. Ackermans	Eindhoven
Professor G. Poots	Hull
Dr. J. Aitchison Dr. A. Crowley Dr. A. Fitt Professor P. Parks	RMCS, Shrivenham
Dr. A. Halton	Buckingham
Dr. J. Gunson Dr. J. Bowcock Dr. G. MacCauley Professor N. Everitt Dr. R. Jones	Birmingham
Dr. A. Wheeler	Bristol
Professor S. McKee Professor D. Owens	Strathclyde
Dr. A. Lacey	Heriot-Watt
Dr. E. Hinch Dr. H. Huppert	Cambridge
Dr. W. Green Dr. T. Rogers Dr. A. England Dr. D. Parker Dr. S. Hibberd	Nottingham
Dr. R. Mackay	Warwick
Professor C. Elliot	Sussex
Dr. M. Baines	Reading

APPENDIX 3

Some Publications Relevant to Study Group Problems

D. Phil. Thesis

T1 *Fitt, V.* Mass Transfer from Spreading Liquid Films, 1984

T2 *Lund, I.* Hydrodynamics and Mass Transfer Problems in Wet Spinning, 1985.

T3 *King, J.R.* Mathematical Aspects of Semiconductor Process Modelling, 1986.

Books

B1 *Elliot, C.M. & McKee, S.* Industrial Numerical Analysis, Oxford 1986.

B2 *Tayler, A.B.* Mathematical Models in Applied Mechanics, Oxford, 1986.

Proceedings

P1. Mathematics of Crystal Growth: Special Issue of IMA J. App. Math. 35, 2 (1985).

Papers

1. *Lacey, A.A.* Initial Motion of the Free Boundary for a Nonlinear Diffusion Equation, IMA J. App. Math 31, 113 (1983).

2. *Ockendon, J.R. & Tayler, A.B.* A model for alloy solidification in Free Boundary Problems: Application and Theory, 3, Pitmans (1985).

3. *Di Benedetto, E. & Friedman, A.* The ill-posed Hele-Shaw model and the Stefan problem for supercooled water. Trans. Amer. Math. Soc. 282 (1984), 183.

4. *Elliot, C.M. & Friedman, A.* Analysis of a model of percolation in a gently sloping sandbank (1984), preprint.

5. *Ughi, M.* A Melting Problem with a Mushy Region: Qualitative Properties. IMA J. App. Math. 33, 135 (1984).

6. *Di. Benedetto, E. & Friedman, A.* Bubble growth in porous media (1985), preprint.

7. *Wilmott, P.* A Note on the WKB Method for Difference Equations, IMA J. App. Math. 34, 295 (1985).

8. *Howison, S.D., Lacey, A.A. & Ockendon, J.R.* Singularity Development in Moving Boundary Problems. Q.J. Mech. App. Math. 38, 343 (1985).

9. *Lacey, A.A. & Ockendon, J.R.* Ill-posed free boundary problems. Control & Cybernetics **14**, 275 (1985).

10. *Fitt, V., Ockendon, J.R., Shillor, M.* Counter-current mass transfer . Int. J. Heat Mass Transf. **28**, 753 (1985).

11. *Aitchison, J. & Howison, S.D.* Computation of Hele-Shaw flows with free boundaries. J. Comp. Phys. (1985).

12. *Crowley, A.B.* Numerical Solution of Alloy Solidification Problems Revisited, in Proc. Coll. a Frontières Libres, eds. *A. Bossavit, M. Frèmond, A. Damlamian.* (1985) Pitmans.

13. *Crowley, A.B.* Alloy Solidification Problems, Control & Cybernetics, **14**, 97 (1985).

14. *Howison, S.D.* Cusp development in Hele-Shaw flow with a free surface. SIAM J. App. Math. **46**, 20 (1986).

15. *Howison, S.D.* Bubble growth in porous media and Hele-Shaw cells. Proc. Roy. Soc. Edinburgh, (1986).

16. *Howison, S.D.* Fingering in Hele-Shaw cells. J. Fluid mech. **167**, 439 (1986).

17. *Please, C.P., Wheeler, A.A. & Wilmott, P.* A Mathematical Model of Cliff Blasting. SIAM J. App. Math. (1986), to appear.

18. *Please, C.P., Wheeler, A.A. & Wilmott, P.* A Continuum Theory for the Mechanics of Rock Blasting. Math. Scientist (1986), to appear.

19. *Wilmott, P.* On the motion of a small two-dimensional body submerged beneath surface waves. J. Fluid Mech., to appear.

20. *Crowley, A.B. & Ockendon, J.R.* Modelling Mushy Regions. Applied Sci. Res. (1986), to appear.

21. *Reed, M.B.* Stresses and displacements around a cylindrical cavity in soft rock. IMA J. App. Math. (1986), to appear.

22. *Chadam, J., Howison, S.D. & Ortoleva, P.* Existence and stability for spherical crystals growing in supersaturated solution. IMA J. App. Math. (1987), to appear.

23. *Please, C.P. & King, J.R.* Diffusion of Dopant in Crystalline Silicon; An Asymptotic Analysis. IMA J. App. Math. (1987), to appear.

24 *Elliott, C.M., Herrero, M.A., King, J.R. & Ockendon, J.R.* The Mesa Problem: Diffusion Patterns for $u_t = \nabla (u^m \nabla u)$, $m \to \infty$. IMA J. App. Math. (1987), to appear.

25. *Aitchison, J., Please, C., Shillor, M.* Heat Flux from a moving plate into laminar co- and counter-currents in thin gaps. Mathematical Engineering in Industry, (1987) to appear.

26. *Hood, M.J.* A Uniqueness Problem for Wave Propagation in a Periodic Structure. IMA J. App. Math. **34**, 279 (1985).

MATHEMATICAL STUDY GROUPS WITH INDUSTRY 1987 – 1988

SEVENTH PROGRESS REPORT ON APPLICATIONS OF DIFFERENTIAL EQUATIONS

Mathematical Institute, Oxford.

1. Introduction

This report is appearing earlier than the customary three years after its predecessor because it seems possible that a watershed has been reached in the Study Group's activities. Factors relevant to this are:

(i) the gradual but inexorable growth of the operation as described in previous reports;

(ii) the experiment of holding the meeting outside Oxford for the first time. The 1988 Heriot-Watt meeting was attended by many people new to the Study Group but some industrial workers were unenthusiastic about the idea of having meetings not near the UK centre of gravity. Moreover the meeting produced a severe financial depletion of the Study Group account;

(iii) there has been a tremendous increase in mathematics-in-industry world-wide. In particular the establishment and rapid growth of ECMI* has provided an easy means of coordinating European mathematical activities of Study Group type;

(iv) the cessation of SERC support for post-doctoral work on Study Group problems.

As usual this report details the activities of the Study Groups at the annual meetings and throughout the rest of the year in Section 2 and 3 respectively. The appendices are mostly just factual lists of the participants and relevant publications but it is hoped that appendix 4, contributed by Dr. Sam Howison, shows something of the human side of the activities.

We would again like to acknowledge gratefully the industrial sponsorship provided by CEGB, ICI, Shell, Courtaulds and Plessey and especially for the 2 years' post-doctoral support for Dr. J. Dewynne from SERC. CASE awards have been held by Miss L. Terrill (Courtaulds), Mr. J. Evans (Plessey) and Mr. S. Wilson (Admiralty) and we are also grateful to industry for these.

2. Problems Discussed at Study Group Meetings

1987

Dr. Bergstrøm came from Elkem, Kristiansand with the novel problem of modelling the heat and mass transfer in a continuously consumed paste electrode in an electro-furnace. A two-phase flow model was constructed which

*See booklet available from Dr. A.B. Tayler, Mathematical Institute, Oxford OX1 3LB

incorporated a free boundary between granular and nongranular paste regions; it seemed that some undesirable instability could be caused by the penetration of this free boundary too far down the electrode (ref. (24)).

Dr. Smith (ECRC) returned with the problem of modelling the mechanics and mass transfer properties of a rotary drier for granular material. It was only possible to summarise the mechanisms which could control evaporation and to give some idea of how these mechanisms could couple into a very simple model for the granular flow.

Dr. Nesvadba (Torry Fish Research Station) returned with the 1980 problem of identifying the thermal parameters which characterise frozen fish. It was felt that the method recommended in 1980 should still be best for fish with low fat content (with corresponding sharp phase change characteristics) but that for fatty fish some of the methods for conventional inverse heat conduction problems may be more useful. A survey of these methods was provided.

Dr. Moore (BSC, Teesside) returned with some convergence problems arising when the grid is refined in turbulent flow modelling but it was soon decided that this problem was much more suitable for UCINA.

Dr. Turner (CERL) was interested in the use of a rigid/plastic model for crack propagation. Although this model poses some interesting mathematical questions, its validity compared to an elastic/plastic model was questioned.

Dr. Jones (Unilever) submitted the problem of wake effects in ink jet printing. Because of the mathematical literature available on the subject, the only suggestions made concerned the optimum droplet charging order.

Dr. Lynch returned from ICI with the rock blasting problem originally posed in Melbourne (1984). This time emphasis was laid on the balance between pressure forces and turbulent drag in the borehole gas as it forces its way through the cracks. This leads to an interesting nonlinear diffusion model for the region opened up by the gas. This equation is analysed in the 1988 D.Phil thesis of Mr. Erhie.

Dr. Frost (Shell, Thornton) presented the problem of calculating the stiffness and damping characteristics of a pivoted slider bearing, with the intention of analysing cross-coupling effects in tilting pad bearings in high-speed compressors. The analysis during the meeting was confined to a study of flat pivoted bearings: work continues on this problem and it is the subject of an M.Sc. dissertation (Miss Culligan, 1988).

Dr. Reading (Shell, Thornton) led a discussion on the modelling of grease as a viscous liquid containing a suspension of solid particles. Reference was made to the mathematical literature on emulsions and particulate suspensions.

Dr. Casselton (STC, Taunton) brought the thermistor problem to the Study Group. Some analysis of the heat and current flow in these temperature dependent circuit devices had already been carried out in Oxford; however as a result of discussion at the Study Group the model was modified to describe more accurately the dependence of the electrical conductivity on temperature. A numerical solution was carried out, together with an asymptotic analysis of the steady state. A simplified model involving heat loss only by conduction down the connecting wires was also proposed. ((14, 21, 22, 26).)

Dr. Mann (CERL) was interested in the effect of geometry on the accumulation of corrosive salts in crevices in reactor supports. A model assuming Darcy flow suggested that a crevice which widened at its mouth and tapered towards its dry end would accumulate less salt than a uniform or oppositely tapered crevice. A model involving two phase flow in the crevice was also proposed.

Mr. Wolfenden (Courtaulds) brought a problem concerning the coagulation of polymer threads. The mass transfer aspects of the problem lead to a diffusion problem with two free boundaries; an outer contracting cuticle and an inner coagulation front. A computer program was written to solve this problem as part of an M.Sc. dissertation (Mr. Martland, 1987). The stability of the cuticle and the possibility of explaining 'kinked cuticles' have also been discussed subsequently.

Dr. Lillington (UKAEA) was concerned at the possibility of the zircalloy cladding of fuel rods igniting in the event of a reactor coolant malfunction. The Study Group considered the possibility of a hot spot on the fuel rod spreading significantly and the likelihood of an ignition front travelling along the rod. It was concluded that either event could occur only at temperatures which greatly exceeded the melting point of the fuel rod.

1988

Dr. Banks and Mr. Wolfenden (Courtaulds) submitted the very interesting problem of modelling the processing of tows of crimped fibres and in particular the tow behaviour between sets of driven rollers. Many suggestions were made ranging from the construction of a continuum version of Courtaulds' original discrete 'spring' model for a single fibre to the assessment of which microscopic variables (i.e. those applicable in the scale of the crimping) are most important in determining macroscopic properties.

Dr. Lynch returned from ICI with a request for a more detailed analysis of the elastic stress field set up in the rock of an exploding cliff by gas forcing its way into cracks emanating from the borehole. The fascinating interaction of the gas filled region both with the cliff face and with a neighbouring borehole blast was also considered; this poses challenging problems in the theory of nonlinear diffusion.

Dr. Halvorsen (Elkem) was interested in the validation and further development of the electrode paste model derived in 1987. Particular attention was paid to the possible effects of concentration and temperature variations on the effective viscosity and it was found that only the latter would be important. Attention was also drawn to the possibility of a mathematical analysis of the so-called 'plasticity test' in which a cylindrical paste specimen is allowed to 'slump' under gravity; the deformation is then used to characterise the mechanical properties of the paste.

Partly as a result of the Cranfield conference (see Section 3), Dr. Billany came from BSC (Teesside) with the long-standing problem of predicting the consumption of flux in the continuous casting of steel. Earlier work by Bland had shown that the thickness of the vertical flux layer adjacent to the mold wall was a crucial variable and efforts were concentrated to try to predict this. A formula for the flux consumption was eventually produced which awaits scrutiny by BSC.

Dr. Collins (Pilkington) was interested in the interaction of the feed of batch/flux and the bulk flow in a glass furnace. The batch was modelled as a gravity current and a lubrication model derived, assuming a prescribed temperature gradient in the bulk and constant viscosity. Suggestions were made for the numerical solution of the problem under conditions where the lubrication model was invalid; these however required the use of primitive variables whereas Pilkington currently use derived variables in their numerical schemes. The need for an accurate model of the batch feed and temperature dependent viscosity to be incorporated into the current model was pointed out. Some numerical aspects have been followed up by UCINA.

Mr. Curran (EEB) described the use of an inductively heated porous metal block as an energy efficient gas heater. He posed the question of determining the exit gas temperature for various block designs and heating distributions; a simple heat and mass transfer model was proposed and solved numerically for a rectangular block.

Dr. Duthie (UKAEA) presented a problem about the behaviour of fuel rods and fluid in a reactor core subject to an earth tremor. Assuming irrotational fluid flow in two dimensions (either horizontal or vertical) and with a simple model for rod impacts at their upper free-ends, it was possible to model the situation by a system of differential equations which had wave-like solutions. Some progress was also made with a 'Wiener-Hopf' model for three dimensional flow. The possibility of chaotic rod motion under conditions of rapid impact was demonstrated by a numerical simulation for the case of two tilting rods.

Dr. Duthie also consulted the group about the efficient numerical solution of a complementarity problem which arose from UKAEA's study of impacting rods. The problem was found to be ill-posed and situations with no solutions, a unique solution and infinitely many solutions were all constructed. A program was written which found the unique solution, when it existed.

3. Other Contacts

Follow up and other spin off activities from the Study Group have been so intensive during the past two years that weekly meetings with industry have been held over periods of several months to discuss specific problems individually. The follow up has concerned the Elkem paste, ICI blasting, Courtaulds wet spinning and coagulation and the Admiralty ship slamming problems. Related work has been the study of pyrotechnic combustion for ICI, (this work has been handed over to RMCS Shrivenham, which is the centre for expertise in the area) and discussions with Shell (Thornton) on grease modelling, volatile fluid interactions and the analysis of 'barrel' shocks at the ruptured end of a tube containing high pressure gas. Continuing discussions have been held with Plessey (Caswell) and Micronas (Helsinki) on semiconductor fabrication (with Mr. J. Evans being the second CASE student to work on this project) and with Plessey (Caswell) on the prevention of non-uniformities in stretched optical fibres; this has since been analysed in (11). Further discussions have also been held with Courtaulds about the fluorescent fabric dye problem.

Despite the changes in personnel at STC (Harlow), the problem of unwinding underwater cables remains a regular topic for discussion, but none of the many ideas put forward has yet been carried through successfully. The thermistor problem which originated with STC (Taunton) (where there have also been personnel changes) has undergone substantial mathematical analysis, and it is hoped that the work may in future be relevant to superconductors.

Although neither problem originated at a Study Group, the areas of turbine blade cooling (Rolls Royce) and fluid/solid acoustic interaction for nondestructive testing (Metal Box) have attracted much attention (see (T2, T3, 10) and (7, 20) respectively); they have resulted in simple but useful and interesting models.

Preliminary discussions have been held with British Aerospace (Bristol) on the problem of lightning strikes on largely nonmetallic aircraft wings (see 1980 report) and with T. Lucas on the modelling of extruders for rapidly cooking cereals and proteins.

On a different level, contact with, and recruitment by, Smith Associates (Guildford) has resulted in their sponsorship of graduate scholarships and an annual lecture on mathematics in the real-world. This has been given by Professor Crighton (Cambridge) (1986), Professor Berry (Bristol) (1987) and will be delivered in 1988 by Professor Friedman (Institute for Mathematics and its Applications) on Mathematics in Industry in the U.S.

Members of the Study Group contributed at a 3 day CEGB in-house mathematics-in-industry Study Group (Leatherhead, December 1986), at a one day Crystal Growth Modellers' meeting (Bristol, April 1987) and also at a 2 day meeting on continuous casting organised by Professor Bland at Cranfield (December 1987).

Internationally, several Study Group members have featured quite prominently at meetings in Claremont (January 1987), and its follow up in Strathclyde (August 1988), (P2), at an Oberwolfach workshop (March 1987), and at the first major SASIAM meeting on Mathematical Methods for Industrial Problems (Bari, September 1988).

More traditional Study Group style workshops have been held at Mar del Plata, Argentina (March 1988), Helsinki (May 1988), Madrid (April 1988), RPI (May 1987, 88), CSIRO/UNSW (February 1988); on each occasion one or more of our members has been an active participant.

Other spin-offs from the Study Group have resulted in M.Sc. dissertations on industrial problems. These include Mr. Young (1987), Mr. Wilson (1987), Mr. Corvi (1987), Mr. Martland (1987), Miss Culligan (1988), Mr. Gay (1988), and Mr. Griffith (1988).

Discussion

It will be apparent that the Study Group concept has gained wide acceptance among mathematicians around the world, and also, as mentioned in 1986, among funding agencies in Brussels and the USA. As mentioned in the introduction, it is thus the future evolution of the concept which is now of major concern and this discussion will not concern the fine tuning of the organisation as it has done in the past.

A questionnaire has been circulated to our most regular participants and it is on the basis of the response to this that proposals will need to be made concerning the future of the UK scheme. Two things are quite clear;

1. The finances need to be put on a firmer foundation.
2. Any future recipe must be such that academics can easily find and collaborate on problems in which they have a genuine mathematical interest.

The question to be addressed is how to achieve both these requirements. It might be quite easy to assure 1 if we moved to a more consultancy or "clinic" based style of operation but the need to restrict the mathematics used to problems where results could be guaranteed would risk possible lowering of the mathematical level at which we operate. On the other hand, satisfying 2, as we presently do, makes it difficult for a monetary value to be assigned to our activities.

We must hope that, aided by our questionnaire, we will be able to devise a realistic proposal for achieving these two aims in the not too distant future.

APPENDIX 1

Industrial Participants (with number of attendances)

BSC Teesside	(2)
CERL Leatherhead	(1)
Courtaulds	(2)
Eastern Electricity Board	(1)
ECRC	(1)
Elkem (Norway)	(2)
ICI	(2)
Pilkington	(1)
Shell Thornton	(1)
STC Taunton	(1)
Torry Research Station Aberdeen	(1)
UKAEA Springfield	(1)
UKAEA Winfrith	(1)
Unilever	(1)

APPENDIX 2

Academic Participants from Institutions Outside the UK

Professor J. Bell	Buffalo, USA
Mr. H. Biesenbach	Bonn, W. Germany
Professor E. Cumberbatch	Claremont, USA
Professor H. Engl	Linz, Austria
Dr. N. Fowkes	Perth, Australia
Dr. A. Head	CSIRO, Clayton, Australia
Professor A. Kapila	RPI, USA
Professor D. Larson	Chevron, USA
Dr. A. Mikelic	Zagreb, Yugoslavia
Dr. S. Rienstra	Nijmegen, Netherlands
Professor Tan Yongji	Fudan, Shanghai, China
Professor E. Varley	Lehigh, USA
Professor R. Westbrook	Calgary, Canada
Mr. W. Xie	Suzhou, China

Academic Participants from Institutions within the UK

Dr. J. Aitchison Dr. A. Crowley Dr. A. Fitt	RMSC, Shrivenham
Dr. W.A. Green Dr. A. King Dr. M. O'Neill Dr. T. Rogers	Nottingham

Dr. E.J. Hinch	DAMTP, Cambridge
Dr. J. Lister	
Dr. H. Stone	
Mr. A. Woods	
Dr. G. Worster	
Dr. S. Bramley	Strathclyde
Dr. R. Gribben	
Dr. R. Hunt	
Professor D. Owens	
Dr. S. Cowley	Imperial College
Dr. M. Shillor	
Professor D. Spence	
Dr. P. Wilmott	
Dr. R. Fletcher	Dundee
Dr. J. Byatt-Smith	Edinburgh
Dr. R. Grundy	St. Andrews
Professor P. Chatwin	Brunel
Professor C. Elliott	Sussex
Dr. C. Please	Southampton
Dr. A. Wheeler	Bristol

APPENDIX 3

Publications Related to Study Group Activities

D. Phil. Theses

T1 *Erhie, H.* Mathematics of Rock Blasting, 1988.

T2 *Morland, L.C.* Mathematical Models for a Fluid Flow arising in Turbine Blade Cooling Passages, 1988.

T3 *O'Malley, K.* An Experimental and Theoretical Study of Slot Injection Separation, 1988.

Proceedings

P1 Proceedings of ESMI: 2 (Oberwolfach, 1987) ed. *H. Neunzert, Teubner* (1988).

P2 Proceedings of ESMI: 3 (Strathclyde, 1988), ed. *H. Neunzert,* To appear.

Papers

1. *King, J.R.* High Concentration Arsenic Diffusion in Crystalline Silicon; An Asymptotic Analysis, IMA J. Appl. Math, **38** (1987).
2. *Lacey, A.A. & Shillor M.* Electrochemical and Electro-Discharge Machining with a Threshold Current, IMA J. Appl. Math, **39** (1987).
3. *Smith, S.A.* Congruent Harmonic and Space Charge Electrostatic Fields, IMA J. Appl. Math, **39** (1987).
4. *Budd, C.J. & Wheeler, A.A.* A New Approach to the Space Charge Problem, Proc. Roy. Soc, **A417** (1988).
5. *Budd, C.J, & Wheeler, A.A.* Exact Solutions of the Space Charge Equation using the Hodograph Method, IMA J. Appl. Math **40** (1988).
6. *Howison, S.D.* Complex Variables in Industrial Mathematics, in Proceedings of the ESMI, 2, ed. H. Neunzert, Teubner (1988).
7. *Tew, R.H., Ockendon, J.R. & Briggs, G.A.D.* Acoustical Scattering by a Shallow Surface-Breaking Crack in an Elastic Solid under Light Fluid Loading. In 'Recent Developments in Surface Acoustic Waves', eds. D.F. Parker & G.A. Maugin, Springer Verlag (1988).
8. *King, J.R.* Phosphorus Diffusion in Silicon, IMA J. Appl. Math. (1989), to appear.
9. *King, J.R.* Extremely High Concentration Dopant Diffusion in Silicon, IMA J. Appl. Math. (1989), to appear.

10. *Dewynne, J. N., Howison, S.D., Ockendon, J.R., Morland, L.C. & Watson, E.J.* Slot Suction from Inviscid Channel Flow, J. Fluid Mech. (1988), to appear.

11. *Dewynne, J.N., Ockendon, J.R. & Wilmott, P.* A Note on a Mathematical Model for Fiber Tapering, SIAM J. Appl. Math. (1988), to appear.

12. *Howison, S.D.* Similarity Solutions to the Stefan Problem and the Binary Alloy Problem, IMA J. Appl. Math. (1988), to appear.

13. *Howison, S.D. & King, J.R.* Explicit Solutions to Six Free Boundary Problems in Fluid Flow and Diffusion, IMA J. Appl. Math. (1989), to appear.

14. *Fowler, A.C., Hinch, E.J. & Howison, S.D.* Temperature Surges in Current Limiting Circuit Devices, to appear in *Proceedings ESMI* 3, ed. H. Neunzert (1989).

15. *Budd, C.J., Friedman, A., McLeod, J.B. & Wheeler, A.A.* The Space Charge Problem, Submitted to SIAM J. Math. Analysis.

16. *Budd, C.J., & Wheeler, A.A.* A Numerical Scheme for the Solution of the Space Charge Problem on a Multiply Connected Region, submitted to J. Comp. Phys.

17. *Budd, C.J., & Wheeler, A.A.* Modelling Coronas and Space Charge Phenomena, to appear in Proc. of ECMI 3 (1988).

18. *Dewynne, J.N., Howison, S.D., Ockendon, J.R. and Xie, W.Q.* Asymptotic Behaviour of Solutions of the Stefan Problem with a Kinetic Condition at the Free Boundary, submitted to J. Austral. Math. Soc., B.

19 *Xie, W.Q.* The Stefan Problem with a Kinetic Condition at the Free Boundary, submitted to SIAM J. Math. Anal.

20 *Tew, R.H.* Acoustic Scattering by a Fluid Loaded Inhomogeneous Membrane, to appear in IMA J. Appl. Math.

21. *Howison, S.D., Rodrigues, J.F. & Shillor, M.* The Thermistor Problem, to appear.

22. *Howison, S.D.* A Note on the Thermistor Problem in Two Space Dimensions, to appear in Quart. Appl. Math.

23. *Fasano, A., Primicerio, M., Howison, S.D. & Ockendon, J.R.* Some remarks on the Regularisation of Supercooled 1-Phase Stefan Problems in One Dimension, to appear in Quart. Appl. Math.

24. *Bergstrøm, T., Cowley, S., Fowler, A.C. & Seward, P.E.* Segregation of Carbon Paste in a Smelting Electrode, IMA. J. Appl. Math. (1989), to appear.

25. *Norbury, J., Cullen, M.J.P., Purser, R.J. & Shutts, G.J.* Modelling the quasi-equilibrium dynamics of the atmosphere, Q. J. Roy. Met. Soc. **113** 735-757 (1987).

26. *Cimatti, G.* A bound on the Solution of the Thermistor Equation, IMA J. Appl. Math. **40** (1988).

27. *Westbrook, R.* The Thermistor: a problem in heat and current flow, submitted to Num. Meth. P.D.E.

28. *Cimatti, G.* Weak Solutions of the Space Charge Problem. IMA J. Appl. Math. (1989), to appear.

29. *Stojanovic, S.* Injection of Ideal Fluid from a Slot into a Stream, IMA J. Appl. Math. (1988), to appear.

30. *Fitt, A., Parker, D.* Non-diffusive Models for Pressure Swing Adsorption, IMA J. Appl. Math. (1988), to appear.

31. *Please, C.P. & Wilmott, P.* The Deposition and Resuspension of Small Radioactive Particles in a Recirculating Flow in a Reactor, Math. Eng. Ind. **1** (1987).

32. *Jeppson, K.O., Anderson, D., Amaratunga, G. & Please, C.P.* Analytical Modelling of Nonlinear Diffusion of Arsenic in Silicon. J. Electrochem. Soc. **134** (1987).

33. *Please, C.P. & King, J.R.* 1- and 2-Dimensional Nonlinear Dopant Diffusion in Crystalline Silicon—Some Analytical Results, Solid State Electronics **31** (1988).

34. *O'Neill, A.G., Hill, C., King, J.R. & Please, C.P.* A New Model for the Diffusion of Arsenic in Polycrystalline Silicon, J. Appl. Phys. **64** (1988).

APPENDIX 4

When did I say that?

In the heated discussions at Study Group meetings, people sometimes make statements that, on reflection, they wish had never crossed their lips; some of them are even committed to paper. Our spies are everywhere, and we have collected a few choice examples. To spare the authors' blushes, they are presented anonymously, but we can vouch for their authenticity.

1. The solid mass of injected fluid . . . .
2. Consider a rigid pulsating cylinder . . . .
3. The hyperbola has a straight bit.
4. $\frac{dy}{dx} = e^x$ . . . . looks pretty nonlinear to me.
5. There is only one unique solution.
6. C is bounded with an unbounded bound.
7. Before you perceive any motion, everything is moving.
8. Just gimme the parameter; I don't care if it's wrong, I'll use it.
9. I have a cubic equation which may well have 3 roots.
10. There is undoubtedly some sort of quasi-reaction going on.
11. Liquid water is present at overwhelming concentration
12. The stagnation point may go half way to infinity then stop.
13. . . . . a long thin spherical region . . . .
14. Most materials are either compressible or incompressible.
15. I don't think it will make the slightest difference, all it will do is change the numbers.
16. There is a parameter in the problem which is probably large or small.
17. You can always make infinity smaller by multiplying by h.
18. There are big ones and some very infinitesimal ones.
19. You can do the inversion completely, analytically, virtually.
20. Consider the rigid elastic-plastic limit.
21. This steady state problem is solved in real time.
22. There is no fudging in this comparison, just a little tuning.

23. The first equation gives 1 = 2; what does this mean for the second equation?