Haptic Touchscreens

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Background. The development of haptic touchscreens, that is touchscreens with the ability to mimic the feel of a real keyboard, would be a significant advance in the mobile phone and tablet market. This report investigates one possible route to developing such touchscreens using piezoelectric actuators placed at the edge of the screen and driven at a range of frequencies.

Scope. The report addresses the following questions: Can an array of piezoelectric transducers placed round the edge of the screen create localised vibrations in the right physiological range to produce the sensation of a keyboard? If so how many transducers are needed? Where should they be placed? How should they be driven? We consider both the excitation of longitudinal and transverse waves but do not discuss surface waves.

Methods. We first consider the full inverse problem in which a key shaped vibration is to be constructed by driving a touchscreen embedded in a phone. We argue that a useful subproblem is to construct a localised vibration in a one dimensional beam driven at one end. If such solutions do exist, that would (1) provide strong evidence that solutions to the full inverse problem do exist, (2) act as good initial starting points towards solutions of the full inverse problem.

Results. We show that in the case of longitudinal waves the mathematical formalism set up for sonar can be exploited. In the case of transverse waves the system is highly dispersive and so a ready-made formalism does not exist. However we show that localised solutions can likewise be constructed.

Conclusions and Recommendations. Our results suggest that it is indeed possible to construct virtual keyboards by driving the edges of a touchscreen. The minimum number of transducers needed is two although we expect that using more would increase resolution. However, before a practical implementation can be made there is still a great deal of work to be done both on the simplified models and on the full problem.

I. INTRODUCTION

A major innovation in mobile phones and mobile computing devices has been the introduction of capacitive touch screens. Touch screens allow the entire area of a phone to act as a display and for custom, context sensitive, keyboards to be used. One key disadvantage of touchscreens is the lack of tactile feedback they provide. This makes touch typing impossible and increases the frequency of typing mistakes.

One possible solution to combining the advantages of a touchscreen with a physical keyboard is the use of haptics to create a virtual keyboard. This report investigates the feasibility of generating virtual keyboards on touchscreen phones by exciting localised vibrations using piezoelectric transducers placed at the edges of the screen.

The current state of the art in haptic feedback on touchscreen devices, available in the latest generation of touchscreen phones, offers macroscopic feedback on key presses by the whole phone vibrating, known as *kinaesthetic* haptic feedback. This is energy inefficient and does not provide any information about whether the key was hit in the centre or close to an edge. Instead what is required is *cutaneous* haptic feedback in which the nerves embedded in the skin are triggered by mechanical forcing.

This report investigates the mathematics needed to create a virtual keyboard via cutaneous haptic feedback. The questions addressed by the report are

- Can an array of piezoelectric transducers surrounding a touchscreen be driven in such a way as to reproduce the feeling of a keyboard?
- If so, how many transducers are needed and where should they be placed?
- To what extent does the finger modify the mechanical properties of the touchscreen?

The subsequent sections of the report are summarised below. In section II we discuss the evidence demonstrating that virtual keyboards are desirable and the physiological constraints on the frequencies and amplitudes of the waves that can be used to mimic keyboards. We then discuss in section III an inverse modelling approach to this problem. The full inverse model is computationally intensive and not suitable for the study group format. We therefore turn to simplified solutions in a less complex geometry. In section IV we argue that the problem of demonstrating feasibility can be reduced to demonstrating that a time and space localised disturbance can be synthesised in one dimension. In section V we analyse the problem with transverse waves using the Kirchhoff plate equation. Section VI describes how results developed

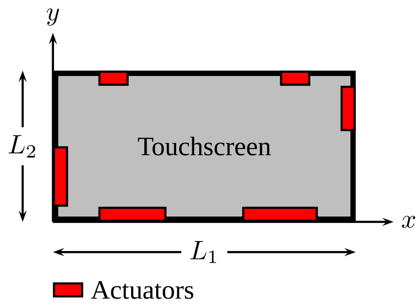

FIG. 1. Geometry of the touchscreen and the transducers. The transducers must be placed at the edge of the touchscreen but can be placed anywhere along the boundary and can be any size.

for sonar can be applied if longitudinal waves are excited. In section VII we discuss the results achieved so far and their limitations. Conclusions are given in section VIII and the future work needed to turn this feasibility study into a practical result is outlined in section IX.

II. HAPTIC TOUCHSCREENS

Research shows that haptic feedback makes a user's interaction with a device more intuitive, productive, faster, accurate, satisfying and engaging[1].

There are physiological and technical constraints on the types of waves that can be generated and used for this application. In order to be detectable displacements must be approximately $u_0 = 30\mu$m in amplitude. The lowest frequency component of the vibration at the fingertip must be in the range 20-500 Hz. The delay between the action (key press) and the response (vibration) must be less than $\Delta t = 100\,\mathrm{ms}$. The actuators must be placed round the edge of the touchscreen as illustrated in figure 1.

III. INVERSE MODELLING

The problem addressed in this report: namely generating a desired output—a localised vibration in the shape of a key on a keyboard—by choosing appropriate inputs—number and location of piezoelectric transducers, and driving waveforms—belongs to the class of problems known as inverse problems. This is in contrast to the more conventional forward modelling where the output is calculated from given inputs. A good discussion of inverse modelling and the challenges usually encountered in inverse modelling problems, such as ill-posedness or instability of the equations, can be found in ref 2.

A working forward model is usually a perquisite for inverse modelling. In a full inverse modelling approach to this problem the forward model would consist of a full finite element simulation of the entire phone, with detailed representations of the touchscreen and transducers but also with some modelling of the mechanical properties of the rest of the phone. Such a model would have to be implemented in a commercial finite element package such as ANSYS.

The complexity of the full inverse modelling approach makes it unattractive since it will be very computationally intensive. The inverse modelling will add an additional layer of complication to the already challenging forward model problem. However, it may be that using the full inverse model will be essential.

The effect we are trying to create—the reinforcement of waves released at different times, at multiple frequencies from several transducers—is very sensitive to errors. The simplified models we discuss below will inevitably introduce small errors. In other contexts these errors would be of little consequence but, for example, a small error in the calculation of wavespeed as a function of frequency could reduce coherence, causing an unacceptable degree of blurring in the vibration of the screen. Despite this, simple models are useful tools in demonstrating feasibility.

IV. SIMPLE MODELS

The main aim of this report is to determine whether the concept of generating a virtual keyboard on a touchscreen by driving its edges with piezoelectric transducers is feasible. For this purpose a very detailed mechanical model of the entire phone is unnecessary. Therefore, we focus on simpler models. We argue that if the approach can be shown to work in simple models, then applying it in more complex cases will also be possible. Furthermore the parameters (waveforms) calculated for simple cases will be good starting points for simulations of more complex and realistic cases. We therefore model just the touchscreen, driven at its edges, and take the limiting case in which the touchscreen can be considered a thin plate obeying the equations of linear elasticity.

Furthermore we argue that in a linear material a waveform of arbitrary complexity can be constructed by superposing localised solutions (called the Green's function approach in the theory of differential equations). Mathematically $g(x,t) \approx g(x,t) \otimes \delta'(x,t)$ where the operator $\otimes$ represents convolution

$$g(x,t) \approx \int_{\infty}^{\infty} \int_{\infty}^{\infty} g(x_1,t_1)\, \delta'(x-x_1, t-t_1)\; \mathrm{d}x_1\, \mathrm{d}t_1 \quad (1)$$

and δ' is a function localised at $x = t = 0$ and with unit integral. The approximation is exact in the limit in which δ' is a product of Dirac delta functions

$$\delta'(x,t) = \delta(x)\, \delta(t) \quad (2)$$

Therefore the problem we focus on is the creation of a localised 'spike' at a desired time and place on the screen.

The final simplification we make is to concentrate on the one dimensional case. Figure 2 illustrates the rationale behind this. As the figure shows, localised solutions in one dimension can be used to construct localised solutions in two dimensions, from either plane waves generated by long transducers (part (a) of the figure), or from circular waves (part (b))

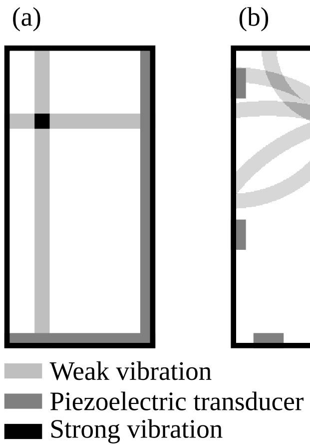

FIG. 2. If localised solutions can be constructed in one dimension, these can be used to construct localised solutions in two dimensions.

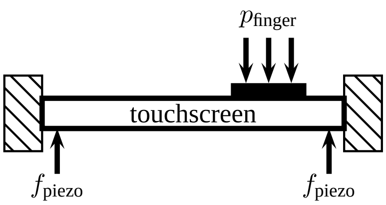

FIG. 3. Schematic of the thin plate model used.

of the figure.

To summarise, if we can show that localised (in time and space) solutions can be generated in one dimension, this provides good evidence that arbitrary waveforms can be constructed in two dimensions.

The mathematical structure of the elastic equations is strongly dependent on the types of waves we choose to excite. A thin elastic sheet supports longitudinal, transverse and surface waves. In this report we focus only on longitudinal and transverse waves. If we consider longitudinal waves, the relevant equation is the non-dispersive wave equation. This allows us to use existing mathematical methods for sonar. Alternatively, if we work with transverse waves the correct equation is the Kirchhoff plate equation, which has solutions similar to the Schrödinger equation allowing the use of mathematics designed to work with quantum mechanical wavepackets.

V. TRANSVERSE WAVES

In this report we consider exciting two different types of wave in the touchscreen: transverse waves and longitudinal waves. The case of longitudinal waves is discussed in the next section. Here we focus on transverse waves, in which the displacements associated with the vibration are perpendicular to the surface of the touchscreen.

A first attempt at modelling the system is shown in figure 3. The touchscreen is modelled as an elastic plate bonded at the edges, with piezoelectric transducers applying forces close to the edges. The finger is modelled as a pressure p_{finger} applied to the surface.

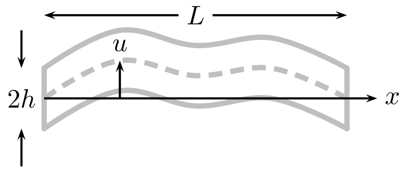

FIG. 4. System described by equation 3.

E	Young's modulus	$70\,\mathrm{GPa}$
ν	Poisson's ratio	$\frac{1}{4}$
ρ	Density	$2500\,\mathrm{kg\,m^{-3}}$
p_{finger}	Finger pressure	$10^4\,\mathrm{Pa} = 1\,\mathrm{N/\,cm^2}$
h	Glass half thickness	$0.5\,\mathrm{mm}$
u_0	Required displacement	$50\,\mu$m
ω_0	Frequency scale	3×10^6 Hz
L	Screen width	$0.1\,\mathrm{m}$

TABLE I. Values of parameters in equation 3.

The simplest model describing transverse waves on a thin plate is the Kirchhoff Plate equation [3]

$$\rho\frac{\partial^2 u}{\partial t^2} + \frac{Eh^2}{3\,(1-\nu^2)}\nabla^4 u = -\frac{p_{\text{finger}}}{2h}, \tag{3}$$

where ρ is the density of the material making up the plate (in $\mathrm{kg\,m^{-3}}$), E is the Young's modulus or stiffness, ν is the Poisson's ratio, h is the half thickness of the plate, p_{finger} is the pressure exerted by a finger, and $\nabla^4 = (\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})^2$ in two space dimensions and $\nabla^4 = \frac{\partial^4}{\partial x^4}$ in one. The terms in equation 3 are illustrated in figure 4 and numerical values of parameters summarised in table I.

The choice of boundary conditions used in this work is illustrated in figure 5. The touchscreen is clamped at its edges by the dust seal and so the 'true' boundary conditions should be

$$u = \frac{\partial u}{\partial n} = 0$$

where $\partial u/\partial n$ is the normal derivative. With these boundary conditions the forcing by the piezoelectric transducers should be included as an external pressure on the right hand side of equation 3. However, as figure 5 illustrates, a good approximation is to keep $u = 0$ at the boundary but to subsume the effect of the piezoelectric transducers into the normal derivative. Thus the boundary conditions used in this work are

$$u = 0 \qquad \frac{\partial u}{\partial n} = \text{forcing}$$

where the forcing is assumed to be applied by the transducers.

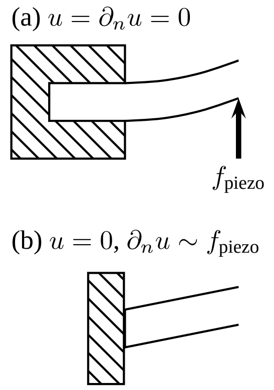

FIG. 5. Choice of boundary conditions. (a) True boundary conditions. (b) Approximate boundary conditions used in this work.

I.e. we include the effect of the transducers by specifying $\partial_n u$ at the boundary as a function of time.

We can simplify the Kirchhoff equation by non-dimensionalisation. We scale position, $\mathbf{x}$, with the size of the system $\mathbf{x} = L\mathbf{x}'$, where $\mathbf{x}' \in [0, 1]^2$. Displacement, u, is scaled by $u = u_0 u'$ where $u_0 = 50\,\mu\text{m}$ is determined by the sensitivity threshold of the finger. To scale time we use a typical frequency $t = t'/\omega_0$, discussed in more detail below.

In dimensionless units

$$\frac{\partial^2 u'}{\partial t'^2} + \lambda^4 \nabla'^4 u' = -\epsilon \tag{4}$$

where

$$\lambda^4 = \frac{Eh^2}{3\,(1-\nu^2)\,L^4 \rho \omega_0^2} \tag{5}$$

$$\epsilon = \frac{p_{\text{finger}}}{2\rho h u_0 \omega_0^2} \tag{6}$$

To estimate the relevant frequency scale we consider the dispersion relation of the dimensional equation. Substituting

$$u = \exp(-i\omega t + ikx)$$

into

$$\rho u_{tt} + \frac{Eh^2}{3\,(1-\nu^2)} \nabla^4 u = 0$$

we obtain

$$\omega^2 = \frac{Eh^2}{3\rho(1-\nu^2)} k^4 \tag{7}$$

In order to resolve details on a scale of $1\,\text{mm}$ we need $k_0 = 2\pi/1\,\text{mm}$ corresponding to a frequency of about $\omega_0 = 3\,\text{MHz}$ which we take as our frequency scale in the non-dimensionalisation.

We can now calculate values of our dimensionless parameters

$$\epsilon = \frac{p_{\text{finger}}}{2\rho h u_0 \omega_0^2} \approx 10^{-5} \ll 1, \tag{8}$$

indicating that the displacement of the screen by pressure exerted by the finger is small compared to the threshold of human sensitivity.

We also have

$$\lambda^4 \approx 10^{-8} \tag{9}$$

$$\lambda \approx 10^{-2} \ll 1 \tag{10}$$

This simply restates that in order to resolve small details we need to excite waves whose wavelength is small compared to the size of the screen.

Thus we neglect the effect of the finger pressure and focus on developing localised vibrations. In the next three sections we consider three approaches to solving this problem.

- *Complete Polynomials* the first approach attempts to construct the desired solution from a complete set of polynomials. This method has problems with generating a solution respecting causality.
- *Focusing Methods* this exploits the high degree of dispersion within the beam equation to focus waves at a desired point. This approach is successful but uses equations only valid in the geometrical optics limit.
- *Image method* this uses an approach similar to the method of images in electrostatics. It is expected to be more accurate than the focusing method but has not yet been fully implemented.

A. Complete Polynomials Approach

The beam equation (the Kirchhoff equation in one dimension) has a complete set of solutions [4] satisfying the boundary conditions $u(-1) = u'(-1) = u(1) = u'(1) = 0$. This is similar to the wave equation which has the Fourier series as a complete set of solutions.

The solutions are of the form

$$C_m(x) = \frac{\cosh \lambda_m x}{\cosh \lambda_m} - \frac{\cos \lambda_m x}{\cos \lambda_m}$$

and

$$S_m(x) = \frac{\sinh \lambda_m x}{\sinh \lambda_m} - \frac{\sin \lambda_m x}{\sin \lambda_m}.$$

We can construct a localised disturbance at a desired location and time from a linear combination of the odd basis functions. The solution satisfies $u(0) = u(1) = u'(1) = 0$. We can read off $u'(0, t)$ and use this as the forcing.

However this solution will not respect the causality condition. We require the localised vibration to be set up in a time $\Delta t < 100\,\text{ms}$, representing the acceptable lag between the

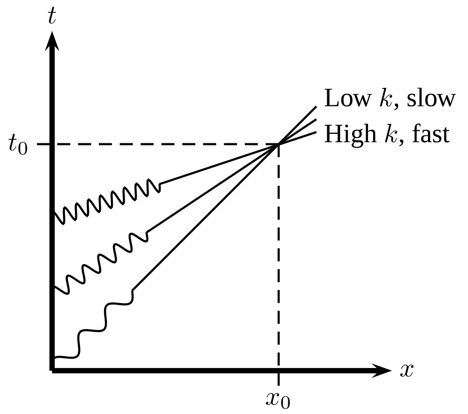

FIG. 6. Ray theory approach to constructing a localised vibration. The medium is dispersive so rays released from a single transducer can be focused on a point.

key being pressed and haptic feedback occurring. The solution generated by the complete polynomials requires u to be non-zero for all time—i.e. we would have to begin driving the edge transducers before the finger press.

Therefore solutions based on this approach are not suitable for our problem.

B. Focusing Approach

In a non-dispersive medium, i.e. one in which $\omega \propto k$, all waves travel at the same speed and constructive reinforcement of waves requires waves from several sources which can be focused on a point. Media described by the Kirchhoff or beam equation are dispersive, $\omega \propto k^2$ (equation 7), meaning that waves with different wavelengths travel at different speeds. This means that waves released from the same point at different times can meet and reinforce. Thus waves released from a single transducer can be focused on a single point, as illustrated in Figure 6.

In the ray theory limit we write

$$u \sim A(x,t)\, e^{i\phi(x,t)/\epsilon}$$

where $\epsilon \ll 1$. Taking partial derivatives

$$u_t \sim \frac{i\phi_t}{\epsilon} u + O(1) \tag{11}$$

$$u_x \sim \frac{i\phi_x}{\epsilon} u + O(1) \tag{12}$$

Where $\omega = -\phi_t$ and $k = \phi_x$. Locally plane waves with ω and k related by the dispersion relation.

The ray tracing equation for uniform media is

$$\phi_{xt} = \phi_{tx} \tag{13}$$

$$\implies \frac{\partial k}{\partial t} + \frac{\partial \omega}{\partial x} = 0 \tag{14}$$

$$\implies \frac{\partial k}{\partial t} + \frac{\mathrm{d}\omega}{\mathrm{d}k}\frac{\partial k}{\partial x} = 0 \tag{15}$$

where the group velocity c_g is given by

$$\frac{\mathrm{d}\omega}{\mathrm{d}k} = c_g.$$

Thus

$$\frac{\partial k}{\partial t} + c_g \frac{\partial k}{\partial x} = 0$$

which means that k is constant along characteristics curves (straight lines in the x, t planes) which in this case can be interpreted as rays

$$\frac{\mathrm{d}x}{\mathrm{d}t} = c_g.$$

We can use this to calculate $\phi(t)$ needed for focusing at x_0, t_0.

$$x - x_0 = 2\lambda^2 k\,(t - t_0)$$

Solve for k and taking $x = 0$, the location from which the waves are generated,

$$k = -\frac{x_0}{2\lambda^2\,(t - t_0)}$$

$$-\phi_t = \omega = \lambda^2 k^2 = \frac{x_0^2}{4\,(t_0 - t)^2\,\lambda^2}$$

$$\phi = \frac{x_0^2}{4\,(t - t_0)\,\lambda^2}$$

Figure 7 shows the output of a simulation to focus a pulse at $x_0 = 0.3$ at $t_0 = 40$ with $\lambda = 0.001$ by imposing the derivative at the left hand boundary to be

$$u_x = \begin{cases} 1000 \sin\left(\frac{x_0^2}{4\lambda^2(t_0 - t)}\right) & \text{for } 0 \le t \le 39.5, \\ 0 & \text{for } t > 39.5. \end{cases} \tag{16}$$

The cut-off at $t = 39.5$ was introduced to avoid exciting unresolvably short waves when $t_0 - t$ is small. The spatial derivatives were approximated using a 5-point centred finite difference stencil on a grid with 64000 points. Boundary conditions on the derivatives were imposed using a 3-point stencil centred on the boundary nodes to supply values for the ghost points just outside the domain. The resulting ODE system was integrated using the fourth-order Runge Kutta method with timestep $\Delta t = \Delta x^2/(4\lambda^2)$.

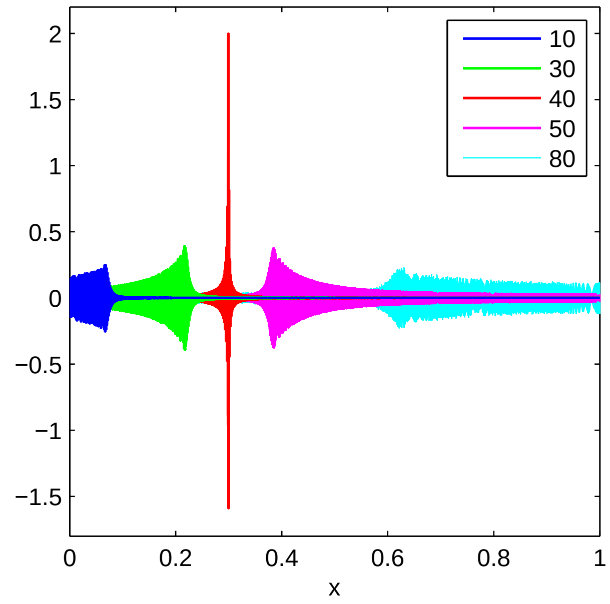

FIG. 7. u plotted against x for various values of t for waves focused at $x_0 = 30$ at time $t_0 = 40$ using ray theory.

This shows that the method works. The signal to noise ratio is relatively low: the signal (localised spike) is four times higher than background noise. This is because the ray theory limit is not completely appropriate for this system. (One additional complication is that the theory takes no account of reflections from the opposite boundary.) In the next section we describe an approach that should allow this limitation to be transcended.

C. Image Method

The image method approach exploits an analogy between the Beam and Kirchhoff equation and the one and two dimensional forms of the Schrödinger equation of quantum mechanics.

To see this we factorise the beam equation.

$$u_{tt} + \lambda^4 u_{xxxx} = \left(\partial_t + i\lambda^2\partial_x^2\right)\left(\partial_t - i\lambda^2\partial_x^2\right) u$$

This shows that solutions to the Schrödinger equation of a particle in free space

$$i\partial_t u = -\lambda^2 \partial_x^2 u \tag{17}$$

and its complex conjugate

$$i\partial_t u = \lambda^2 \partial_x^2 u \tag{18}$$

are also solutions to the beam equation

$$u_{tt} + \lambda^4 u_{xxxx} = 0 \tag{19}$$

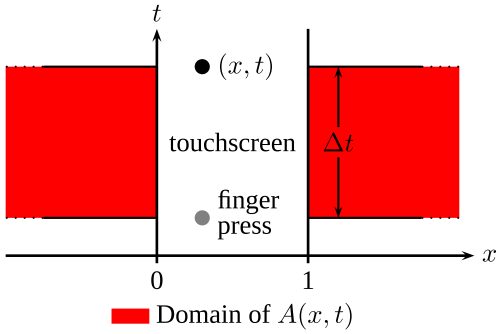

FIG. 8. Domain of integration of equation 21. By causality A can be non-zero only at times between the finger press at $(x, t - \Delta t)$ and the response at (x, t). Furthermore, A must be zero for $0 < x < 1$ i.e. on the touchscreen.

One solution with particularly attractive properties is the localised single particle solution

$$G(x, t - t_0) = \frac{1}{\sqrt{i\,(t - t_0)}} \exp\left[-\frac{x^2}{4i\lambda^2\,(t - t_0)}\right] H(t - t_0) \tag{20}$$

where $H(t)$ is the Heaviside function. This solution is attractive since, as the presence of the Heaviside function shows, causality is built into it. The linearity of the medium means that a valid solution to the beam equation can be constructed using $G(x, t)$ as a building block.

$$u(x,t) = \iint \left[A(x_0, t_0)\, G(x - x_0, t - t_0) + \text{c.c.}\right] \mathrm{d}x_0\, \mathrm{d}t_0 \tag{21}$$

where c.c. stands for complex conjugate, this ensures the resulting $u(x, t)$ is real. The domain of integration is discussed in more detail below.

To apply this approach to our problem $A(x_0, t_0)$ must be chosen so that:

- $u(0) = u(1) = 0$ in order to satisfy the boundary conditions.
- $A(x_0, t_0 < -\Delta t) = 0$ in order to satisfy causality
- $A(0 < x_0 < 1, t_0) = 0$ no transducers within the screen.

These conditions are illustrated in figure 8.

The problem reduces to finding $A(x_0, t_0)$ satisfying these constraints and giving a localised solution. This could be solved for instance by discretising A and solving the problem using computational linear algebra techniques, although a more elegant solution involving integral transforms may be possible. Due to time constraints it was not possible to complete this over the duration of the study group.

Note that this approach can easily be genralised to two dimensions. In this case we have

$$G(x,t) = \frac{1}{\sqrt{it}} \exp\left[-\frac{x^2 + y^2}{4i\lambda^2 t}\right] H(t) \tag{22}$$

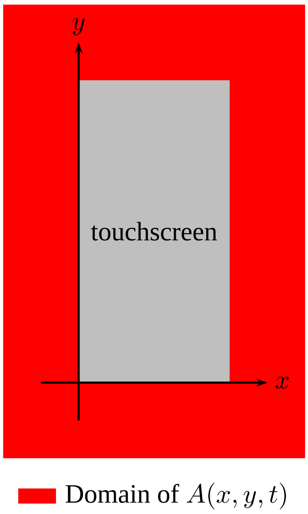

FIG. 9. Spatial part of the domain of A. As before, A can only take non-zero values outside the touchscreen.

$$u(x,t) = \iiint \left[A(x_0, y_0, t_0)\, G(x - x_0, y - y_0, t - t_0) + \text{c.c.}\right] \mathrm{d}x_0\, \mathrm{d}y_0\, \mathrm{d}t_0 \quad (23)$$

The spatial part of the domain of A is illustrated in figure 9, since there are no transducers under the touchscreen A can only take non-zero values outside the touchscreen. As before, by causality $A(x, y, t) = 0$ for $t < -Deltat$.

VI. LONGITUDINAL WAVES

Adopting a beam forming approach to the Haptic problem applies a versatile form of spatial filtering to signals from sensor/actuators located around the edge of a clamped plate. Beamforming spatial filtering separates signals that have different locations but similar frequency content. In the following discussion beamforming and beamsteering will be used in the sense that the objective is to detect and locate a disturbance signal caused by touching the plate at a particular point. Because the system is essentially linear, the issue of responding to such a disturbance by injecting acoustic energy at the point of the disturbance can be managed by inverting the processes used to detect the disturbance. In what follows only the detection problem is discussed on the assumption that the response problem is equivalent.

A. Beamsteering

This beamsteering approach assumes that the detection and injection of signals is fully digitised and that the detection of an input is managed using digital signal processing techniques. It is assumed that the system is essentially linear so

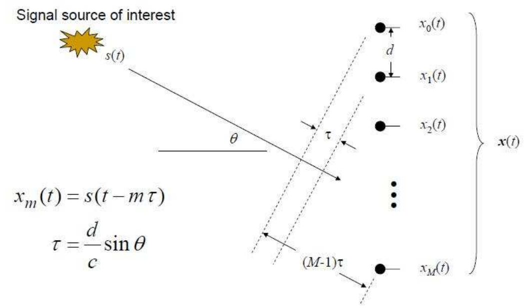

FIG. 10. Array steering.

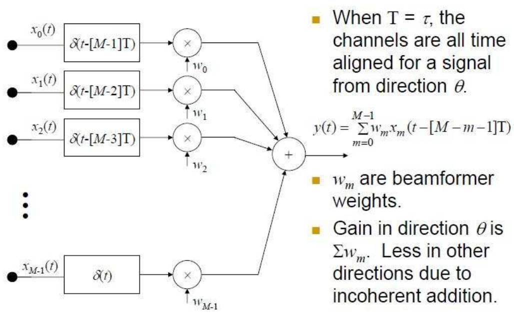

FIG. 11. Time delay addition.

that the processes used to detect an input touch can be inverted to inject acoustic energy to create a stimulus at the detection site. There are plenty of documents describing how to beamsteer in a time-sampled domain to ensure that

- Signals from each sensor/actuator are sampled continuously and stored for time delay processing.
- Signals originating from a specific point on the monitored plane (plate) are added, with appropriate delays, to reinforce wanted features.
- Weights can be applied to the time series samples to minimise or reject signals originating from other areas of the plane.
- Modern digital computers are sufficiently powerful to sample signals from multiple sensors at many multiples of the Nyquist criterion. This allows all the observation points of interest on the plane to be observed in simultaneously.
- Arrays of sensors need not be line arrays. Partially filled windows and randomly distributed arrays work equally well so long as the location of each sensor is known accurately.
- With unlimited signal processing power the technology can be made proof against reflections from the edges of the glass and multimoding caused by internal reflections from the top surface to the bottom surface.

- Many or most of the weighting functions and delay functions can be pre-calculated and stored, thus reducing the need for computing power.

For more information see refs [5, 6].

B. Issues

1. *Near field effects*

Signals from arrays of detectors can be added and processed linearly provided the targets for detection and for injection of acoustic energy are in the far field of the array. This can be guaranteed by using a suitably high frequency to drive the piezo transducers and modulating that carrier with suitable Haptic frequencies to achieve stimulus. Demodulating the carrier at the point of finger touch will have to rely on the non-linear properties of the finger-glass interface.

2. *Size of sensor/actuators*

If a high frequency carrier is used with modulation of the waveform to produce the low frequency effects desired, the wavelength of the vibration is quite small (10MHz implies a wavelength of the order of 0.1mm). This means that the sensors can be quite small, or that the extent of the sensors over multiple wavelengths means that they will sense and emit directionally (i.e. they will not illuminate the plate uniformly. This can be accounted for by adjusting array weights.

3. *Surface Waves vs Body waves*

We have been advised by Analog that the human finger is equally sensitive to stimulus along the finger and across the finger. The current evaluation by Analog uses vertical vibration of the plate as a stimulus (along the finger). It would be worth investigating the use of horizontal vibration in the plate as this would allow the use of existing technology from Surface Acoustic Wave (SAW) devices. SAW technology is mature. The sensors and actuators can be created by deposition on the glass surface. They are physically small. There may be opportunities to use other SAW technology to steer energy and to filter signals. SAW sensors have seen relatively modest commercial success to date, but are commonly commercially available for some applications such as touchscreen displays.

4. *Reflections and Multi-moding*

The problems of multimoding caused by reflections from the edges of the plate and internal reflections between the top and the bottom of the plate will need to be managed. These problems are well understood in the sonar community as sonar signals (active and passive) regularly experience multi-moding from reflections from the sea floor and from sea boundaries and from internal reflections between layers of oceanic water.

5. *No of sensors vs No of points monitored*

An intuitive response is that there should be more sensors than points monitored in the plate. This is not necessarily correct. A partially filled array can have quite high directivity depending on the overall extent of the array. Partially filling the array (using a smaller number of sensors) will result in higher sidelobes (more ambiguity in discriminating between detections). However in this case where the signals to be detected are large and the locations of these disturbances are predefined it should be possible to get away with fewer detectors.

6. *Calculation of sensor input weights*

Calculating the optimal weights to apply to the vectors of sampled observations for each sensed point is straightforward. They are usually derived by a process of inversion from the desired spatial description of the points where lobes and nulls are required.

7. *Signal attenuation*

The Kirchhoff model for vertical vibrations in the plate exhibits three features which cause problems in developing a haptic piezoelectric solution. First the signals are highly dispersive with velocity of propagation very dependent on the frequency of the wave being propagated. Second, the attenuation of the waves in the plate appears extreme. Third the solution developed has low dynamic range and the peaks of energy developed at a point in the plane where the waves cohere are not much greater than the noise level created by the incoherent propagating waves.

C. Why use this approach

The technology to use a beamforming approach to spatial filtering is a mature technology. It has been applied in sonar and radar for many years. Modern computers are sufficiently powerful that they can handle the sampling from many sensors at high rates. They can analyse the sampled signals and process them to monitor multiple observation points simultaneously. There are many efficient techniques to reduce the computational load for sensing a touch. Once the geometry of the plate and the location of the sensors and the sensed locations is set, delays and weights can be precomputed and stored for real-time use. There is a strong possibility that the SAW transducers can be created using deposition techniques.

VII. DISCUSSION

In this section we discuss the results obtained so far, focusing on progress achieved and issues which still remain to be resolved. We consider the relative advantages and disadvantages of using longitudinal waves and transverse waves, the extent to which we have included the effect of the finger in our models, and the problem of optimal location of transducers.

A. Longitudinal vs. transverse waves

In this report we have investigated using both longitudinal and transverse waves to create localised vibrations. We have demonstrated, at least in principle, that either approach could be used to generate virtual keyboards. Here we discuss the relative advantages and disadvantages of both approaches. We consider the energy requirements of both solutions, the mathematical tractability of the equations and whether the same apparatus could also be used to replace capacitive detection of fingers.

a. Energetics Energy efficiency is a major consideration in smartphones. Therefore if either method requires less energy it has a clear advantage. Energy consumption is expected to scale as $\rho\omega^2 u^2$ where ρ is the density of the glass, ω is the driving frequency and u the magnitude of the displacements. The maximum frequency required for either approach is similar: $3\,\mathrm{MHz}$ for transverse waves and $10\,\mathrm{MHz}$ for longitudinal waves, however because of the quadratic dispersion relation for transverse waves the lower frequency modes will require a lower power consumption. Thus, assuming that similar amplitudes are needed for either method, it seems that an approach based on transverse waves will be slightly more energy efficient. However, this is such an important consideration we recommend a more detailed analysis.

b. Mathematical tractability As has been discussed above the mathematical formalism associated with longitudinal waves is simpler (the wave equation only contains a second order derivative in space, whereas the Kirchhoff and beam equations have fourth order spatial derivatives). Furthermore the case of longitudinal waves has been extensively studied in the context of sonar, whereas less work has been done on the Kirchhoff equation.

c. Touch detection It seems clear that apparatus set up to drive longitudinal waves could also be used to detect the waves set up by finger presses and motion, allowing the capacitive sensors to be eliminated. The highly dispersive nature of the transverse waves makes finding the location of finger presses challenging. Further analysis would be needed to see whether it is possible or not.

B. Finger pressure

In our nondimensionalisation of the Kirchhoff equation we showed that the effect of the pressure of the finger is negligible. However we have not had time to consider the effect of more sophisticated models. A more detailed model of the finger is illustrated in Figure 12. While the force exerted by the finger and its spring constant are probably unimportant, the mass of the finger may be comparable to the mass of the glass beneath it. This requires a more careful investigation.

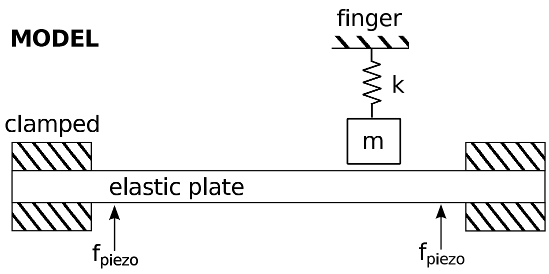

FIG. 12. A more sophisticated finger model.

C. Number and location of transducers

As Figure 2 illustrates, a minimum of two transducers are needed to produce localised vibrations in two dimensions. However only two transducers will produce limited contrast between the non-reinforcing waves and the overlap region. More transducers would produce a greater degree of contrast. A key consideration in placing transducers is locating them so that there is only one region where multiple overlaps occur.

VIII. CONCLUSIONS

In this report we have investigated the possibility of generating virtual keyboards on a touchscreen phone by exciting localised vibrations driven by transducers at the edge of the screen. We show that, although it is likely that inverse modelling using a realistic mechanical model of the entire phone will be needed for quantitative results, we can demonstrate feasibility using simpler models. We have therefore focused on modelling only the touchscreen, idealised as a thin plate. Our results suggest that either longitudinal or transverse waves can be used to generate the virtual keyboards. A minimum of two transducers is needed for this, although using more transducers will improve the signal to background noise ratio.

IX. FURTHER WORK

While the work detailed above provides strong evidence that the construction of virtual keyboards can be generated on a touch screen by driving the edges of the screen there is still a great deal of work to be done before results can be applied to an actual device. Questions which need to be addressed are:

- *Should longitudinal or transverse waves be used?* Our results suggest that both longitudinal and transverse waves can be used to generate virtual keyboards. To decide which is best we should investigate which implementation is likely to consume the least energy and whether the same apparatus can be used both to create virtual keyboards and to detect finger presses.

- *What do the results of simple models suggest?* Simple one and two dimensional models of the touchscreen only should be used in the first instance to determine optimal locations and driving waveforms for transducers, as well as to investigate the effect of the finger in more detail. More realistic models are so complex it is important to get as much done as possible within the framework of the simpler models.

- *Are the results from simple models transferable to realistic models?* The results obtained from simple models should be tested either experimentally or computationally on realistic mechanical models of both the touchscreen and the rest of the phone. It seems likely that it will be necessary to use the results from simple models as the starting point for further simulations rather than as the final results. However, using the results from simple models as a starting point rather than beginning from nothing should save a great deal of time.

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