# Tow Washing Revisited

Courtaulds Plc brought this problem, of washing solvent out of newly spun man-made fibres, to the 1990 Warwick Study Group and again to this one (Wolfenden, 1990a, 1990b).

A "tow" is a flat band 1m wide of one or two million fibres each 0.0015cm diameter, pulled at a speed U of about 1m sec$^{-1}$ through a long bath of solvent, alternately over and under bars called "wedges" (see Figure 1). Byatt-Smith et al (1990) showed how to develop the theory of solvent percolating in and out of the tow (considered as a porous medium), with lubrication theory applied in the thin gap between tow and wedge. They did not, however, consider the viscous boundary layers on the upper and lower surfaces of the tow.

Those boundary layers are the subject of this report. On the upper side of the tow, as it moves from A to C in Figure 1, a first crude approximation will be Blasius's boundary layer for a uniform stream past a flat plate. Fluid is entrained into this with a normal velocity component $0.860(U\nu/x)^{\frac{1}{2}}$ (Rosenhead 1963 p.226) where $\nu$ is the kinematic viscosity and x the distance from A. Clearly this cannot be valid near A, where the wedge modifies the stream outside the boundary layer and Blasius's theory predicts an infinite velocity. The actual flow nearby is presumably a clockwise eddy, ABCDEA whose boundary layer along the wedge wall to A and along the tow to BC should repay detailed investigation; finding the precise speed at which the moving tow sucks in fluid might well involve a triple deck or two. It is, however, clear that the tension along AB will be impeded in its attempt to pull the tow off the wedge at A. It will also be an intriguing problem to locate the dividing streamline between liquid which has come down from E to A and liquid which has been pulled through the tow from below A by the reduced pressure at A.

As the tow approaches C, the liquid which has been entrained in the boundary layer from A is scraped off by the wedge. As a result, (a) the pressure in the liquid between CD and BC will be raised, (b) the two will touch the wedge at C downstream of the point suggested by symmetry, (c) another boundary-layer analysis will be needed. Near D the boundary layer on the wedge presumably separates, making the theory complicated. Near E it may be of the dividing streamline kind analysed by Harper (1963) if the liquid is clean enough, but it is probably sufficiently contaminated for DE to behave like a stationary wall. If so, the boundary layers will separate again near E. However one suspects that then flow may be unstable or even turbulent: the Blasius boundary layer is stable only up to $Ux/\nu$ about 90,000 (Drazin & Reid 1982, p224-6). With $U = 1ms^{-1}$, $\nu = 10^{-6}$ $m^2 s^{-2}$, this needs $x < 9cm$. The distance AC is probably greater than this. It is not surprising that Wolfenden (1990a) found good agreement between experimental and theoretical boundary-layer thickness for $x > 18cm$ with $U = 0.6$ $ms^{-1}$.

To resolve the questions raised here will require much more work on boundary layer theory than can be done within a short study group; it looks more like a thesis problem for a research student.

J.F. Harper.

## References

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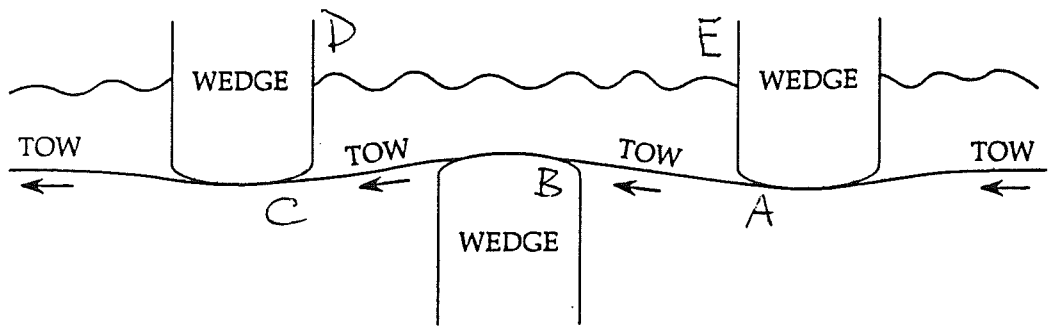
WEDGE

D

E

WEDGE

TOW

TOW

TOW

TOW

C

B

A

WEDGE

Figure 1
(after Byatt-Smith et al. 1990)

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